Section 2.1, selected answers
Math 114 Discrete Mathematics
D Joyce, Spring 2018

2. Use set builder notation to give a description of each of these sets.

a. $\{0, 3, 6, 8, 12\}$.

There are many descriptions of this set. Here's one

$$\{x \mid x \text{ is three times an integer } y \text{ where } 0 \leq y \leq 4\}.$$

b. $\{-3, -2, -1, 0, 1, 2, 3\}$.

One description: $\{x \in \mathbf{Z} \mid z^2 < 10\}$.

c. $\{m, n, o, p\}$.

One description: $\{x \mid x = m \vee x = n \vee x = o \text{ or } x = p\}$.

4. Suppose that $A = \{2, 4, 6\}$, $B = \{2, 6\}$, $C = \{4, 6\}$, and $D = \{4, 6, 8\}$. Determine which of these sets are subsets of which other of these sets.

Of course, each is a subset of itself. But beyond that, B is a subset of A, while C is a subset of both A and D.

26. Suppose that $A \times B = \emptyset$. What can you conclude?

There are various ways to make a conclusion. Here's one. Since the cardinality $|A \times B|$ is 0, either $|A|$ or $|B|$ is 0. So, either A or B is the empty set.

29. If the set A has cardinality m and the set B has cardinality n, then what cardinality does the product $A \times B$ have?

An element of $A \times B$ is an ordered pair (a, b) where a is an element of A and b is an element of B. Since there are m choices for a and there are n choices for b, therefore there are mn choices for the ordered pair (a, b). Thus, $|A \times B|$ is mn.

38. Russell's paradox. Let S be the set $S = \{x \mid x \notin x\}$.

Russell's paradox only works if you have *unrestricted* comprehension. That's what's used in defining S above since there's no restriction on x. After the paradox was discovered, Zermelo restricted comprehension so that the x in the set-builder notation had to be confined to a previously creates set as in $\{x \in X \mid x \notin x\}$ where the set X was already known to exist.

a. Show that the assumption that S is a member of S leads to a contradiction.

Assume $S \in S$. But any element x of S has the property that $x \notin x$. Now, S is one of these elements, so $S \notin S$. That contradicts the assumption $S \in S$.

b. Show that the assumption that S is not a member of S leads to a contradiction.

Assume $S \notin S$. By the definition of S, something x not an element of it must fail to have the property $x \notin x$, that is, if x is not an element of S, then $x \in x$. Therefore, S, not being an element of S, must satisfy $S \in S$. But that contradicts the assumption $S \notin S$.

Conclusion: the existence of the set S, usually called Russell's set, leads to a contradiction. After Russel came up with this contradiction, the axioms of set theory had to be changed to avoid it.

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