Section 3.1, selected answers
Math 114 Discrete Mathematics
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2. Determine which characteristics of an algorithm the following procedures have and which they lack.

An algorithm is a finite set of precise instructions for performing a computation or for solving a problem.

Some properties are listed in the text: input, output, definiteness, correctness, finiteness, effectiveness, and generality.

a. `procedure double(`n`:positive integer)`
    `while` $(n > 0)$
        $n = 2n$

This is simply an infinite loop that has no output. It certainly is a finite set of precise instructions, but it's completely useless since it doesn't solve any problem.

b. `procedure divide(`n`:positive integer)`
    `while` $(n \geq 0)$
    `begin`
        $m = 1/n$
        $n = n - 1$
    `end`

The variable m isn't declared, but other pseudocode examples in the chapter not all variables are declared, so let's assume that's not a problem. There is no explicit output, but the examples of algorithms in the chapter don't have explicit output either. We might take the final value of m to be the output, but that's not interesting, since that final value is 1. That is, if it gets past the previous instruction where the reciprocal of 0 is computed. That could result in a run-time error. The problem with 2b is that it isn't an algorithm to do anything. No problem gets solved. At least it's not an infinite loop like 2a.

c. `procedure sum(`n`:positive integer)`
    $sum = 0$
    `while` $(i < 10)$
        $sum = sum + i$

More bugs. The variable n is never used, and the variable i is never initialized. Note that i is never incremented either, so this is also an infinite loop. We can assume sum was intended to be the output. Since it isn't clear what the problem to be solved is supposed to be, we can't tell to what degree this succeeds or fails to achieve the solution.

d. `procedure choose(`a, b`:integers)`
    $x =$ `either` a `or` b

This fails to be definite because of the choice. As a "nondeterministic" algorithm, it is interesting.

Summary. None of these four fit the basic definition of algorithm since all fail to perform a computation or solve a problem. They all have inputs from specified sets. None have explicit outputs, although 2c and 2d can be interpreted as having outputs. 2a and 2b are definite, but 2c is not because a variable is not initialized, and 2d has the explicit choice. The question of correctness can't even be considered since we don't know what they're supposed to do. Two of them aren't finite, 2a and 2c, but infinite loops instead. Effective? Can each of the steps be performed in a finite amount of time? Sure. The question of generality also can't be considered as we don't know what the purposes are.

12. Describe an algorithm to replace the triple (x, y, z) with (y, z, x). What's the minimum number of assignment statements needed.

Here's one solution. Put x in a temporary variable t; put y in x; put z in y; and put t in x. That is, $t = x; x = y; y = z; z = x$. It takes four assignments.

16. Describe an algorithm that for finding the smallest integer in a finite sequence of natural numbers.

Assume A is a nonempty array indexed from 0 through $n - 1$. This algorithm will return the smallest integer s in the array.

`function smallest(`A, n`: array of natural numbers)`
    $s = A[0]$
    $i = 1$
    `while` $(i < n)$
        `if` $(A[i] < s)$ $s = A[i]$
        $i = i + 1$
    `return` s

19. Describe an algorithm to produce the maximum, median, mean, and minimum of three integers.

There are so many ways to do this that almost any two people will come up with different solutions.

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