Math 114 Discrete Mathematics

Section 8.3, selected answers

D Joyce, Spring 2018

2. Represent each of these relations on the set $\{1, 2, 3, 4\}$ with a matrix.

a. $\{(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)\}$.

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

b. $\{(1, 1), (1, 4), (2, 2), (3, 3), (4, 1)\}$.

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

c. $\{(1, 2), (1, 3), (1, 4), (1, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 4), (4, 1), (4, 2), (4, 3)\}$.

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

d. $\{(2, 4), (3, 1), (3, 2), (3, 4)\}$.

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

5. How can the matrix representing a relation R on a set A be used to determine whether the relation is irreflexive?

Recall that R is irreflexive iff it is not the case that aRa for any element a. That means the diagonal elements in the matrix are all 0.

6. How can the matrix representing a relation R on a set A be used to determine whether the relation is aysmmetric?

Recall that R is asymmetric iff aRb implies $\neg(bRa)$. That means if there's a 1 in the ij entry of the matrix, then there must be a 0 in the ji^{th} entry.

12. How can the matrix for R^{-1}, the inverse of the relation R, be found from the matrix representing R?

Just reflect it across the major diagonal. That is, exchange the ijth entry with the jith entry, for each i and j. The resulting matrix is called the *transpose* of the original matrix.

32. Determine wther the relations represented byt he graphs shown in exercises 26-28 are reflexive, irreflexive, symmetric, antisymmetric, asymmetric, and/or transitive.

For 26. It's reflexive since there's a loop on every vertex. It's not symmetric since there's an arrow from c to d, but there isn't one back. It's not antisymmetric since there are arrows both ways between a and b. Neither is it asymmetric. It's not transitive since $c \to a$ and $a \to b$, but not $c \to b$.

For 27. It's not reflexive since there's no loop at c. It is symmetric since for every arrow, there's an arrow back. It's not transitive since $c \to a$ and $a \to c$, but not $c \to c$.

For 28. It's reflexive, symmetric, and transitive. So, it's an equivalence relation.

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