

Quiz 2
Math 114 Discrete Mathematics
D Joyce, Mar 2018

Scale. 9–10 A, 7–8 B, 4–6 C. Median 8.5.

1. [5] About primes, GCDs, and LCMs. (Show a little bit of work on each part so I know you're not just guessing.)

a. [2] What is the prime factorization of 9100?

9100 is 91 times 100. 100 has the prime factorization $2^2 \cdot 5^2$, and 91 has the prime factorization $7 \cdot 13$. So 9100 has the prime factorization $2^2 \cdot 5^2 \cdot 7 \cdot 13$.

b. [1.5] What is the greatest common divisor of 32 and 100?

Since 32 has the prime factorization $32 = 2^5$ while 100 has the prime factorization $2^2 \cdot 5^2$, therefore their GCD is $2^2 = 4$.

c. [1.5] What is the least common multiple of 32 and 100?

Since 32 has the prime factorization $32 = 2^5$ while 100 has the prime factorization $2^2 \cdot 5^2$, therefore their LCM is $2^5 5^2 = 800$. You can also find it as $\text{LCM} = \frac{32 \cdot 100}{4}$.

2. [3] List the following functions in increasing order of growth so that the slowest growing function comes first, and the fastest growing function last.

$$x, \quad 2^x, \quad \log x, \quad x^2, \quad x(\log x)^2, \quad x^x, \quad \log(\log x)$$

$$\log(\log x) \prec \log x \prec x \prec x(\log x)^2 \prec x^2 \prec 2^x \prec x^x$$

3. [2] Find the smallest value of n so that $(4x^2 + 3x + 1)^3$ is $\mathcal{O}(x^n)$. Explain how you found n in a sentence or two. (You don't have to prove it or find C or k .)

Since $4x^2 + 3x + 1$ is $\mathcal{O}(x^2)$, its cube is $\mathcal{O}(x^6)$.