Name: $\qquad$
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## Math 217/Econ 360 <br> First Test <br> Oct 2014

You may refer to one sheet of notes on this test, the summary table of distributions below, and you may use a calculator. Show your work for credit. You may leave your answers as expressions such as $\binom{8}{4} \frac{e^{1 / 3}}{\sqrt{2 \pi}}$ if you like. Show all your work for credit. Points for each problem are in square brackets.

| Distribution | Mass function $f(x)$ | Mean $\mu$ | Variance $\sigma^{2}$ |
| :---: | :---: | :---: | :---: |
| Bernoulli $(p)$ | $f(0)=q, f(1)=p$ | $p$ | $p q$ |
| Binomial $(n, p)$ | $\binom{n}{x} p^{x} q^{n-x}$, <br> for $x=0,1, \ldots, n$ | $n p$ | $n p q$ |
| Geometric $(p)$ | $q^{x-1} p$, for $x=1,2, \ldots$ | $1 / p$ | $q / p^{2}$ |
| NegativeBinomial $(p, r)$ | $\binom{x-1}{r-1} p^{r} q^{x-r}$, <br> for $x=r, r+1, \ldots$ | $r / p$ | $r q / p^{2}$ |
| Hypergeometric $(N, M, n)$ | $\frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$, <br> for $x=0,1, \ldots, n$ | $n p=M / N$ | $n p q$ |

1. [12] A special deck of 100 cards is numbered from 1 through 100. The deck is shuffled and three cards are dealt. Let $X$ be the first card dealt, $Y$ the second card dealt, and $Z$ the third card dealt. What is the probabilty that $X<Y<Z$ ? Explain your reasoning.
2. [12] An urn contains five red marbles and three green marbles. Two of the eight marbles are chosen at random (without replacement). What is the probability that they are both red? Explain your reasoning.
3. [20] Let $X$ be the number of heads on 5 tosses of a fair coin. a. Fill in this table for the probability mass function $f(x)$ of $X$

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| $f(x)$ |  |  |  |  |  |  |

b. Draw the graph of the cumulative distribution function $F(x)$ of $X$

4. [12] Suppose that $P(A \mid B)=\frac{3}{5}, P(B)=\frac{2}{7}$, and $P(A)=\frac{1}{4}$. Determine $P(B \mid A)$.
5. [13] Prove that if $A$ and $B$ are independent events, then $A$ and $B^{c}$, the the complement of $B$, are also independent events.
6. [18] An experiment can result in one or both of the events $A$ and $B$ with these probabilities:

|  | $A$ | $A^{\mathrm{c}}$ |
| :---: | :--- | :--- |
| $B$ | 0.34 | 0.46 |
| $B^{\mathrm{c}}$ | 0.15 | 0.05 |

Find the following probabilities:
a. $P(A)$.
b. $P(B)$.
c. $P(A \cap B)$.
d. $P(A \cup B)$.
e. $P(A \mid B)$.
f. $P(B \mid A)$.
7. [13] Prove Bayes formula $P(F \mid E)=\frac{P(E \mid F) P(F)}{P(E)}$.

| $\# 1 .[12]$ |  |
| :--- | :--- |
| $\# 2 .[12]$ |  |
| $\# 3 .[20]$ |  |
| $\# 4 .[12]$ |  |
| $\# 5 .[13]$ |  |
| $\# 6 .[18]$ |  |
| $\# 7 .[13]$ |  |
| Total |  |

