Math 217/Econ 360

Sample First Test

Oct 2014

(This sample test is longer than the actual test will be.)

You may refer to one sheet of notes on this test, the summary table of distributions below, and you may use a calculator. Show your work for credit. You may leave your answers as expressions such as $\binom{8}{4}\frac{e^{1/3}}{\sqrt{2\pi}}$ if you like.

Distribution	Mass function $f(x)$	Mean μ	Variance σ^2
Bernoulli(p)	$f(0) = q, f(1) = p$	p	pq
Binomial(n, p)	$\binom{n}{x}p^x q^{n-x}$, for $x = 0, 1, \ldots, n$	np	npq
Geometric(p)	$q^{x-1}p$, for $x = 1, 2, \ldots$	$1/p$	q/p^2
NegativeBinomial(p, r)	$\binom{x-1}{r-1}p^r q^{x-r}$, for $x = r, r+1, \ldots$	r/p	rq/p^2
Hypergeometric(N, M, n)	$\frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$, for $x = 0, 1, \ldots, n$	np $p = M/N$	npq

1. Two fair dice are tossed, one red and one green. What is the probability that they show the same number? Explain your answer in terms of outcomes in a sample space.

2. A fair die is tossed 6 times. What is the probability that exactly three 5's appear?

3. On discrete distributions.

a. Give a simple example of a finite nonuniform discrete real random variable X, that is, give the probabilities of each of its possible values. (You get to choose which one. Don't choose X to be the roll of a fair die since it's uniform.)

b. Determine its expectation $E(X)$.

c. Determine its variance $\mathrm{Var}(X)$.

4. Four cards are dealt from a standard deck of cards (4 suits, 13 cards in each suit). What is the probability that one of the four cards is in each suit (that is, no two of the four cards are in the same suit). Explain your reasoning.

5. Suppose that 5% of males are color blind and 0.25% of females are color blind. A color blind person is chosen at random. What's the probability of this person being male? Assume that there are an equal number of males and females in the entire population. (Be sure to include your computations or explain your reasoning.)

6. On rainy days Pat comes to work on time with probability 0.7, but on nonrainy days he comes to work with probability 0.9. With probability 0.3 it will rain tomorrow.

a. Find the probability that Pat comes to work on time tomorrow.

b. Given that Pat comes to work on time tomorrow, what's the conditional probability that it rained.

7. Suppose you repeatedly roll a pair of fair dice until a pair of 6's appears.

a. What's the probability that the first pair of 6's appears on the 10^{th} roll?

b. What's the expected number of rolls to get the first pair of 6's?

8. Recall that the variance of a random variable X is defined by $\mathrm{Var}(S) = E((X-\mu)^2)$ where $\mu = E(X)$. Using that definition and the properties of expectation, prove that $\mathrm{Var}(X) = E(X^2) - \mu^2$.

9. Suppose a coin has the probability of p of coming up heads. It is flipped repeatedly until either heads or tails has occurred twice. (For example, HTH.) Determine the expected number of flips.