CLARK

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Math 217/Econ 360

Second Test

Nov 2014

You may refer to one sheet of notes on this test, the summary table of distributions below, the table for the normal distribution, and you may use a calculator. Show your work for credit. You may leave your answers as expressions such as $\binom{8}{4}\frac{e^{1/3}}{\sqrt{2\pi}}$ if you like.

1. [10] Let X have the probability density function $f(x)$ and cumulative distribution function $F_X(x)$ given by

$$f_X(x) = \begin{cases} \frac{4}{\pi}\sin^2 x & \text{if } 0 \le x \le \frac{\pi}{2} \\ 0 & \text{otherwise} \end{cases} \qquad F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{2}{\pi}(x - \sin x \cos x) & \text{if } 0 \le x \le \frac{\pi}{2} \\ 1 & \text{if } x > \frac{\pi}{2} \end{cases}$$

Let E be the event $X \le \frac{\pi}{3}$. Determine the probability $P(E)$.

2. [16; 4 points each part] For each of the following functions $F(x)$, could F be a cumulative distribution function for a continuous random variable? You don't have explain your answer; just write "yes" or "no".

________________ **a.** $F(x) = \begin{cases} 0 & \text{if} \quad x \leq 0 \\ x/2 & \text{if} \quad 0 < x < 2 \\ 1 & \text{if} \quad 2 \leq x \end{cases}$

________________ **b.** $F(x) = \begin{cases} 0 & \text{if} \quad x \leq 2 \\ 1/8 & \text{if} \quad 2 < x < 10 \\ 0 & \text{if} \quad 10 \leq x \end{cases}$

________________ **c.** $F(x) = \begin{cases} 0 & \text{if} \quad x \leq 0 \\ 1 - e^{-x} & \text{if} \quad 0 < x \end{cases}$

________________ **d.** $F(x) = \begin{cases} 0 & \text{if} \quad x < 0 \\ \frac{1}{2} & \text{if} \quad 0 \leq x < 1 \\ 1 & \text{if} \quad 1 \leq x \end{cases}$

3. [10] Let X and Y be two independent standard normal distributions. Recall that the probability density function for a standard normal distribution is $\frac{1}{\sqrt{2\pi}} e^{-x^2/2}$. Let $Z = X + Y$. Write down an integral which gives the probability density function $f_Z(z)$ for Z. (Do not evaluate the integral.)

4. [18; 6 points each part] The joint density of X and Y is given by $f(x,y) = xe^{-(x+y)}$ if x and y are both greater than or equal to 0 (and 0 otherwise). Note that $\frac{d}{dy}(-xe^{-(x+y)}) = xe^{-(x+y)}$.

a. Write down a double integral which gives the probability that $X > Y$. (Do not evaluate the integral.)

b. Determine the marginal density function $f_X(x)$ for $x \geq 0$.

c. Determine the conditional density function $f_{Y|X}(y|x)$.

5. [16; 8 points each part] Suppose that we are observing a lump of radioactive plutonium-239. The particular size of the plutonium sample has a rate of 0.4 radioactive emissions per second.

a. What is the probability that the first emission occurs in the first second?

b. What is the probability that the first emission occurs in the second second?

6. [24; 8 points each part] The salaries of physicians in a certain specialty are approximately normally distributed. 25% of these physicians earn less than $180,000 and 25% of them earn more than $320,000.

a. What is the approximate mean μ of the salaries of physicians in this specialty.?

b. What is the approximate standard deviation σ of their salaries?

c. Approximately what percent of these physicians make more than $360,000?

7. [10] The cumulative distribution function for a continuous random variable X is

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ x^2 & \text{if } 0 \le x \le 1 \\ 1 & \text{if } 1 < x \end{cases}$$

Determine the probability density function $f(x)$.

#1.[10]	
#2.[16]	
#3.[10]	
#4.[18]	
#5.[16]	
#6.[24]	
#7.[10]	
Total	

Distribution	Type	Mass/density function $f(x)$	Mean μ	Variance σ^2
$\text{Uniform}(n)$	D	$1/n$, for $x = 1, 2, \ldots, n$	$(n+1)/2$	$(n^2-1)/12$
$\text{Uniform}(a, b)$	C	$\frac{1}{b-a}$, for $x \in [a, b]$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
$\text{Bernoulli}(p)$	D	$f(0) = 1-p, f(1) = p$	p	$p(1-p)$
$\text{Binomial}(n, p)$	D	$\binom{n}{x} p^x (1-p)^{n-x}$, for $x = 0, 1, \ldots, n$	np	npq
$\text{Geometric}(p)$	D	$q^{x-1}p$, for $x = 1, 2, \ldots$	$1/p$	$(1-p)/p^2$
$\text{NegativeBinomial}(p,r)$	D	$\binom{x-1}{r-1} p^r q^{x-r}$, for $x = r, r+1, \ldots$	r/p	$r(1-p)/p^2$
$\text{Hypergeometric}(N,M,n)$	D	$\frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$, for $x = 0, 1, \ldots, n$	np	$np(1-p)$
$\text{Poisson}(\lambda t)$	D	$\frac{1}{x!}(\lambda t)^x e^{-\lambda t}$, for $x = 0, 1, \ldots$	λt	λt
$\text{Exponential}(\lambda)$	C	$\lambda e^{-\lambda x}$, for $x \in [0, \infty)$	$1/\lambda$	$1/\lambda^2$
$\text{Gamma}(\lambda, r)$ $\text{Gamma}(\alpha, \beta)$	C	$\frac{1}{\Gamma(r)} \lambda^r x^{r-1} e^{-\lambda x}$ $= \frac{x^{\alpha-1} e^{-x/\beta}}{\beta^\alpha \Gamma(\alpha)}$, for $x \in [0, \infty)$	r/λ $= \alpha\beta$	r/λ^2 $= a\beta^2$
$\text{Beta}(\alpha, \beta)$	C	$\frac{1}{\mathrm{B}(\alpha, \beta)} x^{\alpha-1}(1-x)^{\beta-1}$, for $0 \leq x \leq 1$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$
$\text{Normal}(\mu, \sigma^2)$	C	$\frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$, for $x \in \mathbf{R}$	μ	σ^2