CLARK UNIVERSITY 1887

Second assignment
Math 217 Probability and Statistics
Prof. D. Joyce, Fall 2014

Assignment on discrete probabilities.

1. (Exercise 3, page 48.) Two dice are thrown. Let E be the event that the sum of the dice is odd, let F be the event that at least one of the dice lands on 1, and let G be the event that the sum is 5. Describe the events $E \cap F$, $E \cup F$, $F \cap G$, $E \cap F^c$, and $E \cap F \cap G$.

2. Let $\Omega = \{a, b, c\}$ be a sample space with $P(a) = \frac{1}{2}$, $P(b) = \frac{1}{3}$, and $P(c) = \frac{1}{6}$. Find the probabilities of all eight subsets of Ω.

3. Give possible sample spaces for each of the following experiments.

(a). An election decides between two candidates A and B.

(b). A two-sided coin is tossed.

(c). A student is asked for the month of the year and the day of the week on which her birthday falls.

(d). A student is chosen randomly from a class of ten students.

(e). You receive a grade in this course.

4. For which of the cases in the previous exercise is it reasonable to assign the uniform distribution function?

5. (Exercise 4, page 48.) Players A, B, and C take turns flipping a coin. The first one to get a head wins. The sample space of this experiment can be defined by

$$\Omega = \{1, 01, 001, 0001, 00001, \ldots\}$$

a. Interpret the sample space.

b. Define the following events in terms of Ω, that is, list the outcomes in each event

(i). the event A for when player A wins
(ii). the event B for when player B wins
(ii). $(A \cup B)^c$

Assume that A flips first, then B, then C, then A, and so on.

6. Describe in words the events specified by the following subsets of

$$\begin{aligned}\Omega &= \{HHH, HHT, HTH, HTT,\\ &\quad THH, THT, TTH, TTT\}\end{aligned}$$

Note that an outcome is the result of flipping three coins (or, if you prefer, flipping one coin three times).

(a). $E = \{HHH, HHT, HTH, HTT\}$.

(b). $E = \{HHH, TTT\}$.

(c). $E = \{HHT, HTH, THH\}$.

(d). $E = \{HHT, HTH, HTT, THH, THT, TTH, TTT\}$.

7. What are the probabilities of the events described in the preceding exercise.

8. Let A and B be events such that $P(A \cap B) = \frac{1}{4}$, $P(A^c) = \frac{1}{3}$, and $P(B) = \frac{1}{2}$. What is $P(A \cup B)$?

9. (Exercise 5, page 48.) A system is composed of 5 components, each of which is either working or failed. Consider an experiment that consists fo observing the status of each component, and let the outcome of the experiment be given by the vector $(x_1, x_2, x_3, x_4, x_5)$, where x_i is equal to 1 if component i is working and is equal to 0 if component i is failed.

a. How many outcomes are in the sample space Ω of this experiment?

b. Suppose the system will work if components 1 and 2 are both working, or if components 3 and 4 are both working, or if components 1, 3, and 5 are all working. Let W be the event that the system will work. Specify all the outcomes in W.

c. Let A be the event that components 4 and 5 are both failed. How many outcomes are contained in the event A.

d. Write out all the outcomes in $A \cap W$.

10. A student must choose exactly two out of three electives: art, French, and mathematics. He chooses art with probability $\frac{5}{8}$, French with probability $\frac{5}{8}$, and art and French together with probability $\frac{1}{4}$. What is the probability that he chooses mathematics? What is the probability that he chooses either art or French?

11. Tversky and Kahneman asked a group of subjects to carry out the following task. They are told that "Linda is 31, single, outspoken, and very bright. She majored in philosophy in college. As a student, she was deeply concerned with racial discrimination and other social issues, and participated in anti-nuclear demonstrations." The subjects are then asked to rank the likelihood of various alternatives, such as

(1) Linda is active in the feminist movement.

(2) Linda is a bank teller.

(3) Linda is a bank teller and active in the feminist movement.

Tversky and Kahneman found that between 85 and 90 percent of the subjects rated alternative (1) most likely, but alternative (3) more likely than alternative (2). Is it? They call this phenomenon the *conjunctive fallacy*, and note that it appears to be unaffected by prior training in probability or statistics. Explain why this is a fallacy. Can you give a possible explanation for the subjects' choices?

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