CLARK UNIVERSITY 1887

Fourth assignment
Math 217 Probability and Statistics
Prof. D. Joyce, Fall 2014

1. A jar has 1000 coins, of which 999 are fair and 1 is double headed. Pick a coin at random, and toss it 10 times. Given that you see 10 heads, what is the probability that the next toss of that coin is also a head?

2. A coin is tossed twice. Consider the following events.

A: Heads on the first toss.
B: Heads on the second toss.
C: The two tosses come out the same.

a. Show that A, B, C are pairwise independent but not independent.

b. Show that C is independent of A and B but not of $A \cap B$.

3. If $P(B^c) = \frac{1}{4}$ and $P(A|B) = \frac{1}{2}$, what is $P(A \cap B)$?

4. Prove that for any three events A, B, C, each having positive probability,

$$P(A \cap B \cap C) = P(A)\, P(B|A)\, P(C|A \cap B).$$

5. Prove that if A and B are independent so are

a. A and B^c.
b. A^c and B^c.

6. If a coin is tossed a sequence of times, what is the probability that the first head will occur after the fifth toss, given that it has not occurred in the first two tosses?

7. A die is rolled until the first time T that a six turns up.

a. What is the probability distribution for T?

b. Find $P(T > 3)$.

c. Find $P(T{>}6 \,|\, T{>}3)$.

8. In London, half of the days have some rain. The weather forecaster is correct $\frac{2}{3}$ of the time, i.e., the probability that it rains, given that she has predicted rain, and the probability that it does not rain, given that she has predicted that it won't rain, are both equal to $\frac{2}{3}$. When rain is forecast, Mr. Pickwick takes his umbrella. When rain is not forecast, he takes it with probability $\frac{1}{3}$.

a. Find the probability that Pickwick has no umbrella, given that it rains.

Suggestion: start by identifying the events. Let

F: the weather forecaster predicts rain
R: it actually rains
U: Mr. Pickwick takes his umbrella

b. Find the probability that he brings his umbrella given that it doesn't rain.

9. Prove that if

$$P(A|C) \geq P(B|C) \text{ and } P(A|C^c) \geq P(B|C^c)\ ,$$

then $P(A) \geq P(B)$.

10. Assume that the random variables X and Y have the joint distribution given here.

		Y			
		-1	0	1	2
X	-1	0	$1/36$	$1/6$	$1/12$
	0	$1/18$	0	$1/18$	0
	1	0	$1/36$	$1/6$	$1/12$
	2	$1/12$	0	$1/12$	$1/6$

a. What is $P(X \geq 1 \text{ and } Y \leq 0)$?

b. What is the conditional probability that $Y \leq 0$ given that $X = 2$?

c. Are X and Y independent?

d. What is the distribution of $Z = XY$?

11. A card is drawn at random from a deck consisting of cards numbered 2 through 10. A player wins 1 dollar if the number on the card is odd and loses 1 dollar if the number if even. What is the expected value of his winnings?

12. In a class there are 20 students: 3 are $5'\,6''$, 5 are $5'\,8''$, 4 are $5'\,10''$, 4 are $6'$, and 4 are $6'\,2''$. A student is chosen at random. What is the student's expected height?

13. In Las Vegas the roulette wheel has a 0 and a 00 and then the numbers 1 to 36 marked on equal slots; the wheel is spun and a ball stops randomly in one slot. When a player bets 1 dollar on a number, he receives 36 dollars if the ball stops on this number, for a net gain of 35 dollars; otherwise, he loses his dollar bet. Find the expected value for his winnings.

14. In a second version of roulette in Las Vegas, a player bets on red or black. Half of the numbers from 1 to 36 are red, and half are black. If a player bets a dollar on black, and if the ball stops on a black number, he gets his dollar back and another dollar. If the ball stops on a red number or on 0 or 00 he loses his dollar. Find the expected winnings for this bet.

Math 217 Home Page at `http://math.clarku.edu/~djoyce/ma217/`