CLARK UNIVERSITY 1887

Sixth assignment answers
Math 217 Probability and Statistics
Prof. D. Joyce, Fall 2014

1. Suppose you choose a real number X from the interval $[2, 10]$ with a density function of the form $f(x) = Cx$, where C is a constant.

a. Find C.

b. Find $P(E)$, where $E = [a, b]$ is a subinterval of $[2, 10]$.

c. Find $P(X > 5)$, $P(X < 7)$, and $P(X^2 - 12X + 35 > 0)$.

2. Suppose you choose a real number X from the interval $[2, 10]$ with a density function of the form $f(x) = C/x$, where C is a constant.

a. Find C.

b. Find $P(E)$, where $E = [a, b]$ is a subinterval of $[2, 10]$.

c. Find $P(X > 5)$, $P(X < 7)$, and $P(X^2 - 12X + 35 > 0)$.

3. Suppose you throw a dart at a circular target of radius 10 inches. Assuming that you hit the target and that the coordinates of the outcomes are chosen at random, find the probability that the dart falls

a. within 2 inches of the center.

b. within 2 inches of the rim.

c. within the first quadrant of the target

d. within the first quadrant of the target and within 2 inches of the center.

4. Suppose you are watching a radioactive source that emits particles at a rate described by the exponential density

$$f(t) = \lambda e^{-\lambda t},$$

where $\lambda = 1$, so that the probability that a particle will appear in the next T seconds is

$$P([0, T]) = \int_0^T e^{-t} dt.$$

Find the probability that a particle (not necessarily the first) will appear

a. within the next second.

b. within the next 3 seconds.

c. between 3 and 4 seconds from now.

d. after 4 seconds from now.

5. Assume that a new light bulb will burn out after t hours, where t is chosen from $[0, \infty)$ with an exponential density

$$f(t) = \lambda e^{-\lambda t}.$$

In this context, λ is often called the *failure rate* of the bulb.

a. Assume that $\lambda = 0.01$, and find the probability that the bulb will *not* burn out before T hours. This probability is often called the *reliability* of the bulb.

b. For what T is the reliability of the bulb equal to $\frac{1}{2}$?

6. Choose independently two numbers B an C at random form the interval $[0, 1]$ with uniform density. Note that the point (B, C) is then chosen at random, uniformly, in the unit square. Find the probability that

a. $B + C < \frac{1}{2}$.

b. $BC < \frac{1}{2}$.

(c). $|B - C| < \frac{1}{2}$.

7. Choose a number U from the unit interval $[0, 1]$ with uniform distribution. Find the cumulative distribution and density for the random variables

a. $Y = U + 2$.

b. $Y = U^3$.

8. Choose a number U from the interval $[0, 1]$ uniformly. Find the cumulative distribution and density for the random variables

a. $Y = 1/(U + 1)$.

b. $Y = \log(U + 1)$.

9. Assume that, during each second, a Dartmouth switchboard receives one call with probability 0.01 and no calls with probability 0.99. Use the Poisson approximation to estimate the probability that the operator will miss at most one call if she takes a 5-minute coffee break.

10. A baker blends 600 raisins and 400 chocolate chips into a dough mix and, from this, makes 500 cookies.

a. Find the probability that a randomly picked cookie will have no raisins.

b. Find the probability that a randomly picked cookie will have exactly two chocolate chips.

c. Find the probability that a randomly chosen cookie will have at least two bits (raisins or chips) in it.

Math 217 Home Page at `http://math.clarku.edu/~djoyce/ma217/`