GRE math study group
Linear algebra examples

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Linear algebra is one of the topics covered by the GRE test in mathematics. Here are the questions relating to linear algebra on the sample test.

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3. If V and W are 2-dimensional subspaces of $\mathbf{R}^4$, what are the possible dimensions of the subspace $V \cap W$?

(A) 1 only (B) 2 only (C) 0 and 1 only (D) 0, 1, and 2 only (E) 0, 1, 2, 3, and 4

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12. Let A be a 2×2 matrix for which there is a constant k such that the sum of the entries in each row and each column is k. Which of the following must be an eigenvector of A?

I. $\begin{bmatrix}1\\0\end{bmatrix}$. II. $\begin{bmatrix}0\\1\end{bmatrix}$. III. $\begin{bmatrix}1\\1\end{bmatrix}$.

(A) I only (B) II only (C) III only (D) I and II only (E) I, II, and III

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18. Let V be the real vector space of all real 2×3 matrices, and let W be the real vector space of all real 4×1 column vectors. If T is a linear transformation from V onto W, what is the dimension of the subspace $\{\mathbf{v} \in V : T(\mathbf{v}) = \mathbf{0}\}$?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

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27. Consider the two planes $x + 3y - 2z = 7$ and $2x + y - 3z = 0$ in $\mathbf{R^3}$. Which of the following sets is the intersection of these planes?

(A) $\emptyset$
(B) $\{(0, 3, 1)\}$
(C) $\{(x, y, z) : x = t, y = 3t, z = 7 - 2t, t \in \mathbf{R}\}$
(D) $\{(x, y, z) : x = 7t, y = 3 + t, z = 1 + 5t, t \in \mathbf{R}\}$
(E) $\{(x, y, z) : x - 2y - z = -7\}$

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36. Let M be a 5×5 real matrix. Exactly four of the following five conditions on M are equivalent to each other. Which of the five condtions is equivalent to NONE of the other four?

(A) For any two distinct column vectors $\mathbf{u}$ and $\mathbf{v}$ of M, the set $\{\mathbf{u}, \mathbf{v}\}$ is linearly independent.
(B) The homogeneous system $M\mathbf{x} = \mathbf{0}$ has only the trivial solution.
(C) The system of equations $M\mathbf{x} = \mathbf{b}$ has a unique solution for each real 5×1 column vector $\mathbf{b}$.
(D) The determinant of M is nonzero.
(E) There exists a 5×5 real matrix N such that NM is the 5×5 identity matrix.

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50. Let A be a real 2×2 matrix. Which of the following statements must be true?

I. All of the entries of A^2 are nonnegative.
II. The determinant of A^2 is nonnegative.
III. If A has two distinct eigenvalues, then A^2 has two distinct eigenvalues.

(A) I only (B) II only (C) III only (D) II and III only (E) I, II, and III

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58. Suppose A and B are $n \times n$ invertible matrices, where $n > 1$ and I is the $n \times n$ identity matrix. If A and B are similar matrices, which of the following statements must be true?

I. $A - 2I$ and $B - 2I$ are similar matrices.
II. A and B have the same trace.
III. A^{-1} and B^{-1} are similar matrices.

(A) I only (B) II only (C) III only (D) I and III only (E) I, II, and III

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