Math 105 History of Mathematics

Final Exam

Prof. D. Joyce, December, 2006

On this take-home exam you may consult your notes for the course, the text book, and any other books you like. Do all your own work; don't consult with anyone but me. If you have any questions about the test, email me or stop by during office hours.

Points for each problem are in square brackets. Start your answers to each problem on a separate page. Staple them together before you hand them in.

Problems 1. Essay. [20]

Please think about this topic and make an outline before you begin writing. You will be graded on how well you present your ideas as well as your ideas themselves. Each essay should be relatively short—one to three written pages, or half to one word-processed pages. There should be no fluff in your essays. Make your essays well-structured and your points as clearly as you can.

Be careful to attribute your sources. See Clark's web page on citation and plagiarism at
`http://www.clarku.edu/departments/writingatclark/citation.cfm`
and the links from there for details.

Trigonometry. Trigonometry was part of the mathematical knowledge in several regions: Greece, India, China, Islamic/Arabic, and western Europe. To what degree was the knowledge of trigonometry passed among these cultures? Back up your conclusions with evidence (which may be taken from our text or other sources). Clarify the degree that you are certain of your conclusions, since, in some cases, the evidence is clear, in others, it may be circumstantial.

Problem 2. On motion (kinematics). [16] Recall that in the early 1300s the Merton scholars proved the mean speed theorem that the distance travelled by an object A undergoing constant acceleration is the same as on object B travelling at a constant velocity equal to the average of initial and final velocities of A. Symbolically, A travels the distance $t(v_i + v_f)/2$ where t is the elapsed time, v_i is A's initial velocity, and v_f is A's final velocity.

Assume that an object A starts from rest (so that $v_i = 0$) and undergoes constant acceleration. Divide the time interval into four equal subintervals. Use the mean speed theorem (either in the style of the Merton scholars without variables, or using Oresme's graphical methods, or using modern algebra) to show that the distances covered in each interval will be in the ratio $1 : 3 : 5 : 7$.

Problem 3. [16] A "Chinese remainder problem" was first described in Sun Zi's *Mathematical Manual* in the third century, and complete solutions to problems of this type were given by

Qin Jiushao in the 13th century. Solve the following problem of this type: find an integer N whose remainder on division by 3 is 1, whose remainder on division by 10 is 2, and whose remainder on division by 14 is 5. In modern notation, this would read

$$\begin{aligned} N &\equiv 1 \pmod 3 \\ N &\equiv 2 \pmod{10} \\ N &\equiv 5 \pmod{14} \end{aligned}$$

Problem 4. [16] Although Eratosthenes was the first person to come at all close to the correct circumference of the earth (as we discussed in the course, see page 99, and he apparently came to within 10 percent of being correct), many others afterwards have tried to measure the earth. One was Al-Biruni (973–1055). Read problem 31 on page 284 for his method. Do that problem and compare Al-Biruni's result to both Eratosthenes' value and the modern value for the circumference of the earth.

(Warning: if you use a calculator in your computations, make sure that you know whether you're working in degrees or radians.)

Problem 5. [16] On the arithmetic triangle.

a. Describe the arithmetic triangle (sometimes called "Pascal's triangle"). Give a few entries in it.

b. Describe at least two applications for it, or, rather, for its entries.

c. Describe whether or not each of the following cultures used the triangle, and for those that did, explain how it was used: Greek, Chinese, Indian, Arabic/Islamic.

Problem 6. On logarithms. [16] One of the interesting subjects during the renaissance was the "double array" which included one geometric array of numbers and one arithmetic array of numbers. For example:

1	3	9	27	81	243	729	2187	6561
0	1	2	3	4	5	6	7	8

Explain how this double array can be used to simplify the process of multiplication. Illustrate your explanation by multiplying 27 by 243 using the double array.

Burgi produced a table of logarithms using just such a double array. He took as the ratio in the geometric array the number 1.0001. Explain why he used a number so close to 1.