Math 105 History of Mathematics

First Test

Prof. D. Joyce, October, 2006

On this take-home exam you may consult your notes for the course, the text book, and any other books you like. Do all your own work; don't consult with anyone but me. If you have any questions about the test, email me or stop by during office hours.

Points for each problem are in square brackets. Start your answers to each problem on a separate page. Staple them together before you hand them in.

Problems 1 and 2. Essays. [20 points each] Select two of the three topics A, B, and C.

Please think about these topics and make an outline before you begin writing. You will be graded on how well you present your ideas as well as your ideas themselves. Each essay should be relatively short—one to three written pages, or half to one word-processed pages. There should be no fluff in your essays. Make your essays well-structured and your points as clearly as you can.

Be careful to attribute your sources. See Clark's web page on citation and plagiarism at
`http://www.clarku.edu/departments/writingatclark/citation.cfm`
and the links from there for details.

Topic A. Explain the logical structure of the *Elements* (axioms, propositions, proofs). How does this differ from earlier mathematics of Egypt and Babylonia? How can such a logical structure affect the mathematical advances of a civilization?

Topic B. Compare and contrast the arithmetic of the Babylonians with that of the Egyptians. Be sure to mention their numerals, algorithms for the arithmetic operations, and fractions. Illustrate with examples. Don't go into their algebra or geometry for this essay.

Topic C. Aristotle presented four of Zeno's paradoxes: the Dichotomy, the Achilles, the Arrow, and the Stadium. Select one, but only one, of them and write about it. State the paradox as clearly and completely as you can. Explain why it was considered important. Refute the paradox, either as Aristotle did, or as you would from a modern point of view.

Problem 3. [20] On areas of circles.

a. How did the Egyptians and Babylonians compute the areas of circles?

b. What is the problem of "quadrature of circle" that vexed the Greeks so much?

c. It has been claimed that Hippocrates proved that (areas of) circles are proportional to (areas of) squares on their diameters. Why is it unlikely that he could have had a rigorous proof?

d. It is likely that Eudoxus first gave a rigorous proof of that fact. What method would he have used for that proof?

Problem 4. [10] Explain the origins of the sexagesimal system (base 60) used in Mesopotamia. (A one-paragraph answer is sufficient.)

Problem 5. [10] Find the greatest common divisor of the two numbers 11484 and 7902 by using the Euclidean algorithm. (Computations are sufficient, but show your work. An explanation is not necessary.)

Problem 6. [20] On Eudoxus' definition of equality of ratios of magnitudes. Answer each part in a couple of sentences.

a. The Pythagorean philosophy of numbers was summarized by the Pythagorean Philolaus as

> All things which can be known have number; for it is not possible that without number anything can be either conceived or known.

Explain why the discovery of incommensurable magnitudes (such as the side of a square and its diagonal) led to a crisis for the Pythagoreans.

b. Explain in your own words Eudoxus definition of equality of ratios, that is, when is the ratio a:b of two magnitudes of one type equal to the ratio c:d of two magnitudes of (possibly) another type.

c. Using this definition, show that the numeric ratio 3:5 does not equal the ratio 4:6.

d. Explain how this definition resolved the crisis and supported the Pythagorean philosophy.