Math 105 History of Mathematics
Second Test Answers

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Scale. 86–100 A. 70-85 B. 60–79 C. Median 78.

Problem 1. On geometry. [20] Consider the construction given in the *Sulvasutras* for constructing a square equal to the difference of two squares: Let $ABCD$ be the larger square with side equal to a, and let $PQRS$ be the smaller square with side equal to b. Cut off $AK = b$ from AB and draw KL perpendicular to AK intersecting DC in L. With K as center and radius KL, draw an arc meeting AD at M. Then the square on AM is the required square.

Prove that the square on AM equals the difference of the squares $ABCD$ minus $PQRS$.

Refer to the diagram. We are to show that the square on AM is the difference of the squares on AB and PQ. Now $AB = BC = KM$. Also, $PQ = AK$. Then the problem is to show that $AM^2 = KM^2 - AK^2$. But triangle AKM has a right angle at A, so the Pythagorean theorem yields the result.

Problem 2. On simultaneous linear congruences. [25] Solve problem I.1 from the *Shushu jiuzhang*. Find the smallest positive integer N so that the four congruences are all satisfied.

$$
\begin{aligned}
N &\equiv 1 \pmod 1 \qquad (1)\\
N &\equiv 1 \pmod 2 \qquad (2)\\
N &\equiv 3 \pmod 3 \qquad (3)\\
N &\equiv 1 \pmod 4 \qquad (4)
\end{aligned}
$$

Either show your work, or explain how you found the answer.

The first congruence, $N \equiv 1 \pmod 1$, is always true. The second congruence, $N \equiv 1 \pmod 2$, says N is odd, but that follows from the fourth one which says N is 1 more than a multiple of 4. Thus, the 3rd and 4th congruences are enough to solve.

Since 3 and 4 are relatively prime, we're looking for a solution modulo 12. The smallest positive integer will be some integer between 1 and 12. Since $N \equiv 1 \pmod 4$, we know N is one of 1, 5, or 9. But $N \equiv 3 \pmod 3$, so 3 divides N. Hence, $N = 9$.

Problem 3. TRUE/FALSE statements. [30]

a. In the ninth century Al-Mamun established at Baghdad a "House of Wisdom" (Bait al-hikma) comparable to the ancient Museum at Alexandria.

True.

b. The *Chou Pei Suan Ching* is the Chinese equivalent of Euclid's *Elements* and includes axioms, postulates, theorems, and proofs in a style very close to Euclid's.

False. No axioms, no postulates, no theorems, no proofs.

c. The parallel postulate was proven to be independent of the rest of Euclidean geometry by the Arabic geometer al-Khwarizmi.

False. It wasn't proven independent until the 19th century.

d. Aryabhata, an Indian mathematician, expressed his mathematics in verse.

True

e. The Merton theorem of kinematics (also known as the mean speed rule) states that a body moving with constant acceleration will traverse the same distance as a body moving uniformly at its mean speed.

True

f. The problem of "finding 1" studied by the Chinese (and others) is finding the solution x to the congruence $ax \equiv 1 \pmod b$ when a and b are relatively prime numbers.

True

g. Oresme's developed a version of fundametal theorem of calculus that said that if a perpendicular whose length is proportional to the velocity at each instant of a time line, then the area of the resulting region is proportional to the distance travelled.

True

h. Negative numbers were accepted in China and India long before they were accepted by the Europeans.

True

i. Although plane trigonometry was well known in ancient Greece and among the Islamic scholars, spherical trigonometry was not known until Western Europeans discovered it after the 13th century.

False. Spherical trig was known to tha ancient Greeks and Islamic scholars.

j. Omar Khayyam gave geometric solutions to the fourteen kinds of cubic equations. (Fourteen kinds were needed since he did not consider negative numbers as coefficients.)

True.

Problem 4. Essay. [25] **The arithmetic triangle.** (Commonly known as Pascal's triangle.)

Which of the several regions that we've studied—Greece, India, China, Islamic/Arabic, and western Europe—was the arithmetic triangle known? How was it used in those regions which knew it? To what degree was the knowledge of the aritmetic triangle passed among these regions? Back up your conclusions with evidence (which may be taken from our text or other sources, but attribute the sources you use). Clarify the degree that you are certain of your conclusions, since, in some cases, the evidence is clear, in others, it may be circumstantial.

So far as we know, only the ancient Greeks did not know of the arithmetic triangle; the other cultures all knew and used it.

Points to make. China: used in 11th century to find n^{th} roots, triangle displayed. India: combinations as early as the 6^{th} centure B.C.E., Mahavira displayed triangle in 9^{th} century. Islam: Al-Samaw'al about 1150 used them for binomial coefficents, Ibn Mun'im in 13^{th} century used them for combinations. Europe: Jordanus dpsplayed it about 1220, later used in Europe for combinations. (Eventually Pascal connected it to probability in the 1600s and his name was connect to it.)

Transmissions: direct evidence is lacking for the most part.