Assignment 1 answers
Math 105 History of Mathematics
Prof. D. Joyce, Spring 2015

**1.** Page 28, ex. 2: Use Egyptian techniques to multiply 34 by 18 and to divide 93 by 5.

| | |
|---|---|
| 1 | 18 |
| $'2$ | 36 |
| 4 | 72 |
| 8 | 144 |
| 16 | 288 |
| $'32$ | 576 |

Since $34 = 2+32$, therefore $34 \cdot 18 = 36+576 = 612$.

| | |
|---|---|
| 1 | 5 |
| 2 | $10'$ |
| 4 | 20 |
| 8 | 40 |
| 16 | $80'$ |
| $\overline{2}$ | $2\,\overline{2}'$ |
| $\overline{5}$ | 1 |
| $\overline{10}$ | $\overline{2}'$ |

After the first five lines, you've still have $93 - 80 - 10 = 3$ to go. You can halve the first line to get $2\frac{1}{2}$ of that 3 leaving $\frac{1}{2}$ to go. The last two lines take care of that. Since $93 = 80 + 10 + 2\frac{1}{2} + \frac{1}{2}$, the quotient is $16 + 2 + \frac{1}{2} + \frac{1}{10}$, that is, $18\,\overline{2}\,\overline{10}$.

**2.** Page 28, ex. 3: Use Egyptian techniques to multiply $\overline{2}\,\overline{14}$ by $1\,\overline{2}\,\overline{4}$.

There are two ways of doing this depending which of the two numbers you take to be the multiplier and which to be the multiplicand. Here's both.

| | |
|---|---|
| $'1$ | $\overline{2}\,\overline{14}$ |
| $'\overline{2}$ | $\overline{4}\,\overline{28}$ |
| $'\overline{4}$ | $\overline{8}\,\overline{56}$ |

| | |
|---|---|
| 1 | $1\,\overline{2}\,\overline{4}$ |
| $'\overline{2}$ | $\overline{2}\,\overline{14}\,\overline{28}$ |
| $'\overline{14}$ | $\overline{14}\,\overline{28}\,\overline{56}$ |

In either case, add marked terms to get the sum $\overline{2}\,\overline{4}\,\overline{8}\,\overline{14}\,\overline{28}\,\overline{56}$.

Now, that's a fine answer, but it happens to simplify to 1. It's not clear how the Egyptians would have found the simplification, but they often used auxiliary computations. They might have put $\overline{14}\ \overline{28}\ \overline{56}$ in terms of seven $\overline{56}$'s as 'red auxiliaries' and simplified it to $\overline{8}$ as an intermediate computation.

**3.** Page 28, ex. 5: Show that the solution to the problem of dividing 7 loaves among 10 men is that each man gets $\overline{\overline{3}}\,\overline{30}$.

Start with the row 1 10 . You could take $\frac{1}{3}$ of that then twice that to get $\frac{2}{3}$ of it, but the Egyptians would have used a special $\frac{2}{3}$ table for that.

| | |
|---|---|
| 1 | 10 |
| $\overline{\overline{3}}$ | $6\,\overline{\overline{3}}'$ |
| $\overline{10}$ | 1 |
| $\overline{30}$ | $\overline{3}'$ |

Now $6\,\overline{\overline{3}}$ is short of 7 by $\frac{1}{3}$, and the next two rows get you that remaining $\frac{1}{3}$. Since 7 is the sum of the right entries in the second and fourth rows, the left entries in those rows give the answer, namely, $\overline{\overline{3}}\,\overline{30}$.

**4.** Page 28, ex. 7. Multiply the Egypian fractions $7\,\overline{2}\,\overline{4}\,\overline{8}$ by $12\,\overline{\overline{3}}$ using the Egyptian multiplication technique.

| | |
|---|---|
| 1 | $7\,\overline{2}\,\overline{4}\,\overline{8}$ |
| 2 | $15\,\overline{2}\,\overline{4}$ |
| $'4$ | $32\,\overline{2}$ |
| $'8$ | 63 |
| $'\overline{\overline{3}}$ | $4\,\overline{\overline{3}}\,\overline{3}\,\overline{6}\,\overline{12}$ |

Note that $\overline{\overline{3}}$ times 7 is $4\,\overline{\overline{3}}$. The product is the sum of the entries on the right of the marked lines, and that simplifies to $99\,\overline{2}\,\overline{4}$.

**5.** Page 28, ex. 8. Calculate 2 divided by 11 and 2 divided by 23 in the style of the Egyptians.

First, we'll show that 2 divided by 11 is $\overline{6}\,\overline{66}$.

$$\begin{array}{rr}
1 & 11 \\
\overline{\overline{3}} & 7\,\overline{3} \\
\overline{3} & 3\overline{\overline{3}} \\
\overline{6} & 1\,\overline{2}\,\overline{3}' \\
\overline{11} & 1 \\
\overline{66} & \overline{6}' \\
\hline
 & 1\,\overline{2}\,\overline{3}\,\overline{6} \\
 & 2
\end{array}$$

Next, we'll show that 2 divided by 23 is $\overline{12}\,\overline{276}$.

$$\begin{array}{rr}
1 & 23 \\
\overline{\overline{3}} & 15\,\overline{3} \\
\overline{3} & 7\overline{\overline{3}} \\
\overline{6} & 3\,\overline{2}\,\overline{3} \\
\overline{12} & 1\,\overline{\overline{3}}\,\overline{4}' \\
\overline{23} & 1 \\
\overline{276} & \overline{12}' \\
\hline
 & 2
\end{array}$$

**6.** Solve problem 32 of the *Rhind mathematical papyrus*: A quantity, its 1/3, and its 1/4, added together become 2. What is the quantity? Of course, the answer in modern terms is 24/19, but try to come up with the answer in the style of the Egyptians, and express the answer in unit fractions.

There are several approaches that you could take to this problem. One is the method of false position. Take a convenient number as a solution and when that doesn't give the correct answer, make an adjustment. Since we have to divide the quantity by 3 and 4, a convenient number would be 12.

If you take 12, 1/3 of 12, and 1/4 of 12, and add them together, you get $12+4+3 = 19$. We're suppose to get 2, not 19. So to get the correct answer, take our guess, 12, multiply it by 2 to get 24, and divide by 19. That means the correct answer is 24 divided by 19. Thus, we've converted our original question to one where we have to divide 24 by 19.

$$\begin{array}{rr}
1 & 19' \\
\overline{2} & 9\,\overline{2} \\
\overline{4} & 4\,\overline{2}\,\overline{4}' \\
\overline{19} & 1 \\
\overline{38} & \overline{2} \\
\overline{76} & \overline{4}' \\
\hline
 & 24
\end{array}$$

So the answer is $1\,\overline{4}\,\overline{76}$.

**7.** Page 28, ex. 14: Solve problem 11 of the *Moscow Mathematical Papyrus*: The work of a man in logs; the amount of his work is 100 logs of 5 handbreadths diameter; but he has brought them in logs of 4 handbreadths diameter. How many logs of 4 handbreadths diameter are there?

The volume of logs of the same length are in a ratio of the squares of their diameters. Here, that means 16 of the larger 5-handbreadth logs would equal 25 of the smaller 4-handbreadth logs.

We need to find how many of the smaller logs would equal 100 larger logs. So we need to solve the proportion

$$16 : 25 = 100 : \text{unknown}.$$

The Egyptians would have used the "rule of three" to solve the proportion. 100 times 25 divided by 16. That computation gives the answer $156\,\overline{4}$.

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