Assignment 1 answers  
Math 105 History of Mathematics  
Prof. D. Joyce, Spring 2015

1. Page 28, ex. 2: Use Egyptian techniques to multiply 34 by 18 and to divide 93 by 5.

\[
\begin{array}{c|c}
1 & 18 \\
2 & 36 \\
4 & 72 \\
8 & 144 \\
16 & 288 \\
32 & 576 \\
\end{array}
\]

Since 34 = 2 + 32, therefore \(34 \cdot 18 = 36 + 576 = 612\).

After the first five lines, you’ve still have 93 − 80 − 10 = 3 to go. You can halve the first line to get \(2 \frac{1}{2}\) of that 3 leaving \(1 \frac{1}{2}\) to go. The last two lines take care of that. Since 93 = 80 + 10 + 2 \(\frac{1}{2}\) + \(\frac{1}{10}\), the quotient is \(16 + 2 + \frac{1}{2} + \frac{1}{10}\), that is, 18 \(\frac{7}{10}\).

2. Page 28, ex. 3: Use Egyptian techniques to multiply \(\frac{2}{14}\) by \(\frac{1}{27}\).

There are two ways of doing this depending which of the two numbers you take to be the multiplier and which to be the multiplicand. Here’s both.

\[
\begin{array}{c|c}
1 & \frac{2}{14} \\
2 & \frac{4}{28} \\
4 & \frac{8}{56} \\
\end{array}
\]

In either case, add marked terms to get the sum \(2 \frac{4}{14} 14 28 56\).

Now, that’s a fine answer, but it happens to simplify to 1. It’s not clear how the Egyptians would have found the simplification, but they often used auxiliary computations. They might have put \(14 28 56\) in terms of seven \(56\)’s as ‘red auxiliaries’ and simplified it to \(\frac{8}{3}\) as an intermediate computation.

3. Page 28, ex. 5: Show that the solution to the problem of dividing 7 loaves among 10 men is that each man gets \(\frac{3}{30}\).

Start with the row \(1\ 10\). You could take \(\frac{1}{3}\) of that then twice that to get \(\frac{2}{3}\) of it, but the Egyptians would have used a special \(\frac{2}{3}\) table for that.

\[
\begin{array}{c|c|c}
1 & 10 & \\
\frac{3}{5} & 6 \frac{3}{5} & \\
\frac{10}{5} & 1 & \\
\frac{30}{3} & \frac{3}{5} & \\
\end{array}
\]

Now \(6 \frac{3}{5}\) is short of 7 by \(\frac{1}{3}\), and the next two rows get you that remaining \(\frac{1}{3}\). Since 7 is the sum of the right entries in the second and fourth rows, the left entries in those rows give the answer, namely, \(\frac{3}{30}\).

4. Page 28, ex. 7. Multiply the Egyptian fractions \(\frac{7}{2} \frac{2}{14} \frac{4}{8}\) by \(\frac{12}{3}\) using the Egyptian multiplication technique.

\[
\begin{array}{c|c|c|c|c|c|c}
1 & 7 & 2 & 4 & 8 & \\
2 & 15 & 2 & 47 & \\
4 & 32 & 2 & \\
8 & 63 & \\
\frac{3}{5} & 4 & \frac{3}{5} & 6 & 12 & \\
\end{array}
\]

Note that \(\frac{3}{5}\) times 7 is 4 \(\frac{3}{5}\). The product is the sum of the entries on the right of the marked lines, and that simplifies to \(99 \frac{2}{3}\).

5. Page 28, ex. 8. Calculate 2 divided by 11 and 2 divided by 23 in the style of the Egyptians.
First, we’ll show that 2 divided by 11 is $\frac{2}{11}$.

\[
\begin{array}{c|c}
1 & 11 \\
3 & \overline{73} \\
6 & \overline{123} \\
\hline
\text{Total} & \frac{1}{2}
\end{array}
\]

Next, we’ll show that 2 divided by 23 is $\frac{2}{23}$.

\[
\begin{array}{c|c}
1 & 23 \\
3 & 153 \\
6 & 323 \\
\hline
\text{Total} & \frac{12}{12}
\end{array}
\]\n
\[
\begin{array}{c|c}
22 & \frac{1}{2} \\
38 & \frac{2}{2} \\
70 & \frac{4}{4} \\
\hline
\text{Total} & \frac{24}{24}
\end{array}
\]

So the answer is $1\frac{1}{2}$. page 28, ex. 14: Solve problem 11 of the 

Moscow Mathematical Papyrus: The work of a man 
in logs; the amount of his work is 100 logs of 5 hand-
breadths diameter; but he has brought them in logs 
of 4 handbreadths diameter. How many logs of 4 
handbreadths diameter are there?

The volume of logs of the same length are in a 
ratio of the squares of their diameters. Here, that 
means 16 of the larger 5-handbreadth logs would 
equal 25 of the smaller 4-handbreadth logs.

We need to find how many of the smaller logs 
would equal 100 larger logs. So we need to solve 
the proportion

\[
16 : 25 = 100 : \text{unknown}
\]

The Egyptians would have used the “rule of 
three” to solve the proportion. 100 times 25 di-
vided by 16. That computation gives the answer 
156 $\frac{4}{4}$.

6. Solve problem 32 of the Rhind mathematical 
papyrus: A quantity, its 1/3, and its 1/4, added to-
gether become 2. What is the quantity? Of course, 
the answer in modern terms is 24/19, but try to 
come up with the answer in the style of the Egyp-
tians, and express the answer in unit fractions.

There are several approaches that you could take 
to this problem. One is the method of false posi-
tion. Take a convenient number as a solution and 
when that doesn’t give the correct answer, make an 
adjustment. Since we have to divide the quantity 
by 3 and 4, a convenient number would be 12.

If you take 12, 1/3 of 12, and 1/4 of 12, and add 
them together, you get $12 + \frac{4}{3} + \frac{3}{4} = 19$. We’re sup-
pose to get 2, not 19. So to get the correct answer, 
take our guess, 12, multiply it by 2 to get 24, and 
divide by 19. That means the correct answer is 24 
divided by 19. Thus, we’ve converted our original