Math 114 Discrete Math

Prof. D. Joyce

Second test, March 2008

Your name: ____________________________

You may use one sheet of prepared notes during this test. Show all your work for credit. Points for each problem are in square brackets.

Problem 1. On set operations. [12; 4 points each part] Let $A = \{0, 1, 2, 3\}$, $B = \{0, 1, 4, 5\}$, and $C = \{0, 2, 4, 6\}$. Find

a. $A \cup (B \cap C)$.

b. $A \oplus (B - C)$.

c. $(A \cup B \cup C) - (A \cap B \cap C)$.

Problem 2. On algorithms and their complexity. [20; 10 points each part] Choose either the linear search algorithm or the binary search algorithm when you answer this question. Your choice.

To standardize notation, suppose you are looking for the value V among $A_1, A_2, \ldots, A_n$. For the binary search, assume that the array is sorted in increasing order.

a. Describe the algorithm either in English or in pseudocode. Be sure to include the possibility that V is not one of the values $A_1, A_2, \ldots, A_n$.

b. Describe the worst case time complexity of the linear search algorithm, that is, state the order of the time it takes the algorithm to execute ($\mathcal{O}(n), \mathcal{O}(\log n), \mathcal{O}(n \log n)$ or whatever). Then in a sentence or two explain why that's correct.

Problem 3. On the growth of functions. [20] Use the definition for big-$\mathcal{O}$ notation to show that $f(x) = 3x + 4$ is $\mathcal{O}(x)$. (Recall for positive-valued functions that f is $\mathcal{O}(g)$ if there are constants C and k such that $f(x) < Cg(x)$ for $x > k$.)

Problem 4. [28; 4 points each part] True or false. Just write the word "true" or the word "false". If it's not clear to you which it is, explain; otherwise no explanation is necessary.

_______ **a.** If A, B, and C are three sets, then the only way that $A \cup C$ can equal $B \cup C$ is if $A = B$.

_______ **b.** There is no one-to-one correspondence between the set of all positive integers and the set of all odd positive integers because the second set is a proper subset of the first.

_______ **c.** If f is a function $A \to B$, and S and T are subsets of A, then $f(S \cap T) = f(S) \cap f(T)$.

_______ **d.** $\lfloor 14.85 \rfloor + \lceil 14.85 \rceil = 30$.

_______ **e.** $\sum_{k=0}^{10} 7 = 70$.

_______ **f.** If $f(n) = (3n^6 + 5n^2 - 6)(x + \log x)$, then f is $\mathcal{O}(x^6 \log x)$.

_______ **g.** If the product $A \times B$ of two sets A and B is the emptyset $\emptyset$, then both A and B have to be $\emptyset$.

Problem 5. On divisibility. [20] One of the theorems used to prove that Euclid's algorithm is valid is the following:

Theorem. The greatest common divisor of two positive integers a and b, where $a < b$, equals the greatest common divisor of a and $b - a$.

Prove this theorem that $\textsc{Gcd}(a, b) = \textsc{Gcd}(a, b - a)$. You can use the definition of $\textsc{Gcd}(a, b)$ (the largest integer d that divides both a and b) and the definition of divides ($a|b$ iff $\exists c, ac = b$).

#1.[12]	
#2.[20]	
#3.[20]	
#4.[28]	
#5.[20]	
Total	