Your name: ___________________________

Math 114 Discrete Mathematics

First Midterm

February 2018

You may use a calculator and one sheet of prepared notes during this test. Show all your work for credit. Points for each problem are in square brackets.

1. Negating propositions. [15; 5 points each part] For each of the following propositions, write the negation of the proposition so that negations only appear immediately preceding predicates; there should be no negations of conjunctions, disjunctions, or quantifiers.

a. $P(x) \wedge P(y)$

b. $\forall x\, P(x)$

c. $\forall x\, \exists y\, (P(x) \to Q(x, y))$

2. On truth tables. [20; 10 points each part]

a. Use a truth table to determine whether $(p \vee q) \to r$ is logically equivalent to $(p \to r) \wedge (q \to r)$. Explain in a sentence why your truth table says whether they are logically equivalent or not.

p	q	r	
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

b. Use a truth table to determine whether $(p \wedge r \to q) \wedge (q \to p) \to (p \vee \neg q \vee r)$ is a tautology, a contradiction, or a contingent proposition. Explain in a sentence why your truth table shows whether it is a tautology, a contradiction, or a contingent proposition.

p	q	r	
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

3. Interpretation of symbolic expressions. [25; 5 points each part] Determine the truth value of each of the following statements if the universe of discourse for each variable consists of all real numbers. Simply write "true" or "false" for each; no need to explain why.

__________ **a.** $\forall x\,(x^2 \geq 0)$.

__________ **b.** $\exists y\,\forall x\,(x < y)$.

__________ **c.** $\exists x\,\exists y\,(y^2 - x^2 = 1)$.

__________ **d.** $\forall x\,\exists y\,(y^2 - x^2 = 1)$.

__________ **e.** $\exists y\,\forall x\,(y^2 - x^2 = 1)$.

4. On proofs. [15] Prove that for any positive integer n, if 3 divides n^2, then 3 divides n. Here, "divides" means divides without remainder. [Suggestion: one way you can do this is by a proof by contradiction using cases. There are 3 cases. Case a: 3 divides n without remainder (which you're trying to show). Case b: there is a remainder of 1 when 3 divides n. Case c: there is a remainder of 2 when 3 divides n.]

5. On sets. [25] True/false. For each sentence write the whole word "true" or the whole word "false". If it's not clear whether it should be considered true or false, you may explain in a sentence if you prefer.

__________ **a.** If $|A| = |B|$, then $A = B$.
(Recall that $|A|$ is the cardinality of the set A.)

__________ **b.** If $A = \{1, 3, 5, 7, 9\}$, then $|\wp(A)| = 32$.
(Recall that $\wp(A)$ is the powerset of A.)

__________ **c.** If $A \cap B = A$, then $A \cup B = B$.

__________ **d.** $A \cap B \subseteq C$ implies $A \cup B \cup C \subseteq A \cup B$.

__________ **e.** The composition of two one-to-one functions is also a one-to-one function.
(Recall that a one-to-one function is also called an injection.)

#1.[15]	
#2.[20]	
#3.[25]	
#4.[15]	
#5.[25]	
Total	