

Math 114 Discrete Mathematics First Midterm February 2018

You may use a calculator and one sheet of prepared notes during this test. Show all your work for credit. Points for each problem are in square brackets.

- 1. Negating propositions. [15; 5 points each part] For each of the following propositions, write the negation of the proposition so that negations only appear immediately preceding predicates; there should be no negations of conjunctions, disjunctions, or quantifiers.
 - **a.** $P(x) \wedge P(y)$

b. $\forall x P(x)$

c. $\exists x \, \forall y \, (P(x) \to Q(x,y))$

2. On truth tables. [20; 10 points each part]

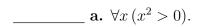
a. Use a truth table to determine whether $(p \land q) \to r$ is logically equivalent to $(p \to r) \lor (q \to r)$. Explain in a sentence why your truth table says whether they are logically equivalent or not.

$\mid p \mid$	q	r
T	T	T
$\mid T$	T	F
$\mid T$	F	T
$\mid T$	F	F
F	T	T
$\mid F \mid$	T	F
F	F	T
$\mid F$	F	F

b. Use a truth table to determine whether $(p \lor q \to r) \to ((p \to r) \land (q \to r))$ is a tautology, a contradiction, or a contingent proposition. Explain in a sentence why your truth table shows whether it is a tautology, a contradiction, or a contingent proposition.

$\overline{}$		
p	q	r
T	T	T
$\mid T \mid$	T	F
$\mid T \mid$	F	T
$\mid T \mid$	F	F
F	T	T
F	T	F
F	F	T
F	F	F

3.	Interpretation of symbolic expr	essions.	[25; 5 pc	oints each p	part] Determ	nine the
tru	th value of each of the following state	ments if t	he univers	se of discou	rse for each	variable
con	sists of all real numbers. Simply write	true" or	"false" fo	or each; no	need to expla	ain why.



_____ **b.**
$$\forall y \, \exists x \, (x < y).$$

_____ **c.**
$$\forall x \,\exists y \,(y^2 - x^2 = 1).$$

_____ **d.**
$$\exists y \, \forall x \, (y^2 - x^2 = 1).$$

_____ e.
$$\exists x \,\exists y \,(y^2 - x^2 = 1)$$
.

4. On proofs. [15] Prove that for any positive integer n, if 3 divides n^2 , then 3 divides n. Here, "divides" means divides without remainder. [Suggestion: one way you can do this is by a proof by contradiction using cases. There are 3 cases. Case a: 3 divides n without remainder (which you're trying to show). Case b: there is a remainder of 1 when 3 divides n. Case c: there is a remainder of 2 when 3 divides n.]

5 .	On se	ets.	[25]	True/false	e. For	each	sentence	write	the	whole	word	"true"	or the	e whole
wor	d "fals	se".	If it's	not clear	wheth	er it	should b	e cons	$ider\epsilon$	ed true	or fal	lse, you	may	explain
in a	sente	nce i	if you	prefer.										

a. If $A = \{1, 3, 5, 7, 9\}$, then $|\mathcal{Q}(A)| = 16$. (Recall that $\mathcal{Q}(A)$ is the powerset of A.)

_____ b. $A \cap B \subseteq C$ implies $A \cup B \cup C \subseteq A \cup B$.

_____ **c.** If $A \cup B = B$, then $A \cap B = A$.

_____d. The composition of two onto functions is also an onto function. (Recall that a onto function is also called a surjection.)

_____ e. If A = B, then |A| = |B|. (Recall that |A| is the cardinality of the set A.)

#1.[15]	
#2.[20]	
#3.[25]	
#4.[15]	
#5.[25]	
Total	