Section 3.6, selected answers
Math 114 Discrete Mathematics
D Joyce, Spring 2018

2. Convert from decimal to binary.

a. 231. Here's an easy way. Repeatedly halve, marking the odd numbers.

231	1
115	1
57	1
28	0
14	0
7	1
3	1
1	1

Then read up to get the binary form: 1110 0111.

3. Convert from binary to decimal.

d. 110 1001 0001 0000. Here's one way. Starting with the left 1, repeatedly double and add the next digit.

$$\begin{aligned} 1 &= 1. \\ 2+1 &= 3. \\ 6+0 &= 6. \\ 12+1 &= 13. \\ 26+0 &= 26. \\ 52+0 &= 52. \\ 104+1 &= 105. \\ 210+0 &= 210. \\ 420+0 &= 420. \\ 840+0 &= 840. \\ 1680+1 &= 1681. \\ 3362+0 &= 3362. \\ 6724+0 &= 6724. \\ 13448+0 &= 13448. \\ 26896+0 &= 26896. \end{aligned}$$

8. Convert each of the following integers from binary notation to hexadecimal notation.

This is easy, and that's the whole reason for using hex in the first place. Each group of 4 bits is named be one hex digit.

a. 1111 0111. Four ones gives the decimal 15, which is the hex digit f, while 0111 gives 7. So f7 is the hex equivalent.

b. 1010 1010 1010. 1010 in base 2 gives the decimal 12, which is c in hex. So ccc is the hex equivalent.

c. 111 0111 0111 0111. These are all 7s. 7777.

24. Use the Euclidean algorithm to find GCDs.

Repeatedly replace the larger by the remainder after dividing by the smaller.

a. $\text{GCD}(1,5) = \text{GCD}(1,0) = 1$.

b. $\text{GCD}(100,101) = \text{GCD}(100,1) = \text{GCD}(0,1) = 1$.

c. $\text{GCD}(123,277) = \text{GCD}(123,31) = \text{GCD}(30,31) = \text{GCD}(30,1) = \text{GCD}(0,1) = 1$.

d. $\text{GCD}(1529,14039) = \text{GCD}(1529,278) = \text{GCD}(139,278) = \text{GCD}(139,0) = 139$.

e. $\text{GCD}(1529,14038) = \text{GCD}(1529,277) = \text{GCD}(144,277) = \text{GCD}(144,133) = \text{GCD}(11,133) = \text{GCD}(11,1) = \text{GCD}(0,1) = 1$.

f. $\text{GCD}(11111,111111) = \text{GCD}(11111,1) = \text{GCD}(0,1) = 1$.

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