Math 114 Discrete Mathematics

Section 5.3, selected answers

D Joyce, Spring 2018

1. List all the permutations of $\{a, b, c\}$.

There are 6 of them: abc, acb, bac, bca, cab, and cba. You can write them in any order, but the order given here is the *lexicographic* order, that is, they're arranged alphabetically.

2. How many permutations are there of the set $\{a, b, c, d, e, f\}$?

There are 6! since the first letter can be any of 6, the next any of 5, etc. In general, there are $n!$ permutations of a set of size n. $(6! = 720.)$

6. Find the numerical values of each of the following quantities.

a. $\binom{5}{1} = 5.$

In general, $\binom{n}{1} = n$; there are n ways to choose 1 element out of a set of n elements.

b. $\binom{5}{3} = \dfrac{5!}{3!\,2!} = \dfrac{5 \cdot 4}{2 \cdot 1} = 10.$

c. $\binom{8}{4} = \dfrac{8!}{4!\,4!} = \dfrac{8 \cdot 7 \cdot 6 \cdot 5}{4 \cdot 3 \cdot 2 \cdot 1} = 70.$

d. $\binom{8}{8} = 1.$

In general, $\binom{n}{n} = 1$; there is only 1 way to choose all n elements out of a set of n elements, and that is to take them all.

e. $\binom{8}{0} = 1.$

In general, $\binom{n}{0} = 1$; there is only 1 way to choose no elements out of a set of n elements, and that is to take none of them.

f. $\binom{12}{6} = \dfrac{12!}{6!\,6!} = \dfrac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 924.$

8. In how many different orders can five runners finish a race if no ties are allowed?

This asks the number of permutations of a set of size 5. Any of the 5 can come in first, any of the remaining 4 second, any of the remaining 3 third, etc. So there are 5! altogether. $(5! = 120.)$

10. There are six different candidates for governor of a state. In how many different orders can the names of the candidates be printed on a ballot.

The number of permutations of a set of size 6 is $6! = 120$.

15. In how many ways can a set of five letters be selected from the English alphabet?

The number of subsets of size 5 taken from a set of size 26 is $\binom{25}{5}$. Numerically, this is

$$\frac{26!}{21!\,5!} = \frac{26 \cdot 25 \cdot 24 \cdot 23 \cdot 22}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 65780.$$

20. How many bit strings of length 10 have

a. exactly three 0s?

Select 3 places in the string of length 10 to put the 0's. There are $\binom{10}{3}$ ways of doing that. Numerically, that's 120.

b. the same number of 0's as 1's?

In other words, 5 each. Select 5 places in the string of length 10 to put the 0s. There are $\binom{10}{5}$ ways of doing that. Numerically, that's 252.

27. A club has 25 members.

a. How many ways are there to choose four members of the club to serve on an executive committee?

How many ways can you choose 4 elements out of a set of 25? $\binom{25}{4}$. Numerically, that's 12650.

b. How many ways are there to choose a president, vice president, secretary, and treasurer of the club?

Assume that no member can hold more than one office. There are 25 choices for president, then 24 remaining choices for vice president, etc. So the answer is $25 \cdot 24 \cdot 23 \cdot 22$, which is 303600.

The difference between parts **a** and **b** is that if you permute the officers, the result is a different slate, but if you permute the committee members, you don't get a different committee.

28. A professor writes 40 discrete mathematics true/false questions. Of the statements in these questions, 17 are true. If the questions can be positioned in any order, how many different answer keys are possible?

Each key is determined by the set of 17 true questions. There are $\binom{40}{17}$ subsets of size 17 taken from a set of size 40. That's approximately $8.87 \cdot 10^{10}$.

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