Math 114 Discrete Mathematics

Section 5.4, selected answers

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2. Find the expansion of $(x+y)^5$

a. using combinatorial reasoning.

$(x+y)^5 = (x+y)(x+y)^4$, and $(x+y)^4$ can be computed as $x^4+4x^3y+6x^2y^2+4xy^3+y^4$, so $(x+y)^5 = (x+y)(x^4+4x^3y+6x^2y^2+4xy^3+y^4)$, which, when multiplied out gives $1x^5+5x^4y+10x^3y^2+10x^2y^3+5xy^4+y^5$.

b. using the binomial theorem.

It's just

$$\binom{5}{0}x^5+\binom{5}{1}x^4y+\binom{5}{2}x^3y^2+\binom{5}{3}x^2y^3+\binom{5}{4}xy^4+\binom{5}{5}y^5$$

which is

$$1x^5+5x^4y+10x^3y^2+10x^2y^3+5xy^4+y^5$$

4. Find the coefficient of x^5y^8 in $(x+y)^{13}$.

It's just $\binom{13}{5}$, which is $\dfrac{13\cdot 12\cdot 11\cdot 10\cdot 9}{\cdot 4\cdot 3\cdot 2\cdot 1}$ which is 1287.

12. The row of Pascal's triangle containing the binomial coefficients $\binom{10}{k}$, where $0\le k\le 10$, is

1 10 45 120 210 252 210 120 45 10 1.

Use Pascal's identity to product the row immediately following this row in Pascal's triangle.

Just start with the number 1, then add adjacent numbers of the preceding row, then end with another 1.

1 10 45 120 210 252 210 120 45 10 1.

1 11 55 165 330 462 462 330 165 55 11 1.

31. Show that a nonempty set has the same number of subsets with an odd number of elements as it does subsets with an even number of elements.

There are many ways that you can prove this. Here's a proof by induction. The base case is when a set has 1 element. It has two subsets, one empty (which has an even number of elements), and one being the whole set (which has an odd number of elements). Thus, the base case holds.

For the inductive step of the proof, let S be a set with n elements. We append one new element z to that set to get a set $T = S\cup\{z\}$ with $n+1$ elements. Each subset of S gives rise to two subsets of T—one that includes z and one that doesn't have z—one of these two sets has an even cardinality and the other an odd cardinality. Thus, the same number of subsets of T has an even cardinality as has an odd cardinality.

(Note that the inductive hypothesis was never invoked. Thus, this proof apparently doesn't depend on the axiom of induction. Actually, it does, indirectly at least, as that axiom is needed to prove properties of evenness and oddness.)

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