Math 114 Discrete Mathematics

Section 6.1, selected answers

D Joyce, Spring 2018

4. What is the probability that a randomly selected day of the year from a possible 366 days is in April?

Thirty days hath September, April, etc. Assuming the probability distribution is uniform, then the probability is $\frac{30}{366}$, which is about 0.08197.

6. What is the probability that a card selected from a deck is an ace or a heart?

There are 4 aces and 13 hearts, and there's 1 ace of hearts. That gives $4 + 13 - 1 = 16$ cards each of which is an ace or a heart. So the probability is $\frac{16}{52}$, which equals $\frac{4}{13}$.

10. What is the probability that a five-card poker hand contains the two of diamonds and the three of spades?

There are $\binom{52}{5}$ possible hands. How many of them have the two of diamonds and the three of spaces? $1 \cdot 1 \cdot \binom{50}{3}$ because there are $\binom{50}{3}$ ways of choosing the remaining 3 cards. So the probability is

$$\binom{50}{3} \Big/ \binom{52}{5} = \frac{50!}{3!\,47!}\,\frac{5!\,47!}{52!} = \frac{5 \cdot 4}{52 \cdot 51} = \frac{5}{663}$$

21. What is the probability that a die never comes up an even number when it is rolled six times?

There are three odd numbers on the die. There are, therefore, 3^6 possible outcomes without even numbers. But there are 6^6 possible outcomes altogether. So the probability is $3^6/6^6 = 1/2^6 = 1/64$.

An easier way to see it that half the outcomes for each roll aren't even, so the probability is $(1/2)^6$.

25. Find the probability of winning the lottery by selecting the correct six integers, where the order in which these integers are selected does not matter, from the positive integers not exceeding

a. 50. There are $\binom{50}{6}$ combinations. Only one is correct; so the probability is $1 \Big/ \binom{50}{6}$. That's 1/15890700, about 0.000000063.

b. 52 . $1 \Big/ \binom{52}{6} = 1/20358520$.

c. 56 . $1 \Big/ \binom{56}{6} = 1/32468436$.

d. 60 . $1 \Big/ \binom{60}{6} = 1/50063860$.

30. What is the prob. that a player wins the prize offered for correctly choosing five (but not six) numbers out of six integers chosen between 1 and 40, inclusive, by a computer?

You need to choose 5 of the correct 6 numbers, and 1 of the incorrect 34 numbers. That gives $5 \cdot 34 = 170$ ways of choosing five but not six numbers correct. There are $\binom{40}{6}$ possible choices altogether. That gives you a probability of $170 \Big/ \binom{40}{6}$.

Math 114 Home Page at `http://math.clarku.edu/~djoyce/ma114/`