Math 114 Discrete Mathematics

Section 8.1, selected answers

D Joyce, Spring 2018

4. Determine whether the relation R on the set of all people is reflexive, symmetric, antisymmetric, and/or transitive, where $(a, b) \in R$ if and only if

a. a is taller than b.

Assuming that "taller" means strictly taller and not the same height, then R is not reflexive, but irreflexive. Also, R is antisymmetric since if a is taller than b, then you know b is shorter, not taller, than a. But R is transitive since if a is taller than b and b is taller than c, then a is taller than c.

b. a and b were born on the same day.

Reflexive since one is born on one's own birthday. Symmetric since if a was born on b's birthday, then so too will b be born on a's birthday. And it's transitive.

c. a has the same first name as b.

Assuming everyone has a first name, then it's reflexive. Also symmetric and transitive.

d. a and b have common grandparents.

Assume this means $\exists g$ such that g is a grandparent of both a and b. Then R is reflexive and symmetric. But not transitive: g can be a grandparent of a and b, while h is a grandparent of b and c, but a and c needn't have a common grandparent.

10. Which relations in exercise 4 are irreflexive?

Just 4a.

17. Which relations in exercise 4 are asymmetric?

The difference between *asymmetric* and *antisymmetric* is a fine point. A relation is *asymmetric* if both of aRb and bRa never happen together. A relation is *antisymmetric* if both of aRb and bRa never happens when $a \neq b$ (but might happen when $a = b$). Thus, any asymmetric relation is antisymmetric, but some antisymmetric relations aren't asymmetric. Warning: other authors may use asymmetric and/or antisymmetric differently than Rosen.

4a is both asymmetric and antisymmetric.

24. Let R be the relation $R = \{(a, b) \mid a < b\}$ on the set of integers. Find

a. R^{-1}. The inverse relation is $\{(a, b) \mid a > b\}$.

b. $\overline{R}$. The complementary relation is

$$\{(a, b) \mid a \not< b\},$$

that is, $\{(a, b) \mid a \geq b\}$.

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