Name: _____

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Math 120 Calculus I

Final Exam Sample

November 2011

You may use a calculator and one sheet of notes. Leave your answers as expressions such as $e^2\sqrt{\dfrac{\sin^2(\pi/6)}{1+\ln 10}}$ if you like. Show all your work for credit.

1. A closed rectangular box with a square base is to be built subject to the following conditions: the volume is to be 27 cubic feet, the area of the base may not exceed 18 square feet, the height of the box may not exceed 4 feet.

a. Express the volume V and the total surface area A of the box in terms of a side s of the base and the height h of the box.

b. Determine the dimensions of the box for minimal surface area. Explain in a sentence how you determined that was the minimum.

2. Let $f(x) = 4x + \dfrac{1}{x}$.

a. Find the critical points of f.

b. Determine the intervals on which f is increasing and the intervals on which f is decreasing.

c. Determine whether each critical point gives a local maximum, local minimum, or neither.

3. Let $f(x) = \frac{1}{2}x^4 + x^3 - 18x^2 + 2x - 15$..

a. For what values of x does f have an inflection point?

b. Determine the intervals on which f is concave up, and the intervals on which it is concave down.

4. Let $f(x) = 4x^{3/2}$.

a. Compute the derivative of f.

b. State the assumptions of the Mean Value Theorem and show that f satisfies those assumptions on the interval $[a, b] = [1, 4]$.

c. State the conclusion of the Mean Value Theorem and find a value in the interval $(1, 4)$ which satisfies that conclusion .

5. Sketch the graph of one function f with the following properties.

(a). f is positive on the intervals $(1, 3)$, $(7, 9)$, and $(11, \infty)$, but it's negative on the intervals $(-\infty, 1)$, $(3, 7)$, and $(9, 11)$.

(b). f' is positive on the intervals $(-\infty, 2)$, $(5, 8)$, and $(10, \infty)$, but it's negative on the intervals $(2, 5)$ and $(8, 10)$.

(c). f'' is positive on the intervals $(4, 6)$ and $(9, \infty)$, but it's negative on the intervals $(-\infty, 4)$ and $(6, 9)$.

Note on your sketch any local maxima, minima, and points of inflection.

6. Compute derivatives of the following functions.

a. $f(x) = \ln(\sin x)$.

b. $f(x) = e^x \tan 2x$.

c. $f(x) = \dfrac{4x^5 - 3x^2 + x - 10}{\cos x + \sec x}$.

7. Compute the following limits.

a. $\lim\limits_{x \to \pi} \dfrac{5 \sin x + 4 \cos x}{2x + e^x}$

b. $\lim\limits_{h \to 0} \dfrac{\sqrt{x+h} - \sqrt{x}}{h}$. (Hint: multiply and divide by a conjugate.)

c. $\lim\limits_{x \to \infty} \dfrac{5x^3 - 2x}{3x^2 + 1}$