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Math 120 Calculus I  
First Test Sample  
October 2011

You may use a calculator. Leave your answers as expressions such as  $e^2 \sqrt{\frac{\sin^2(\pi/6)}{1 + \ln 10}}$  if you like. Show all your work for credit. Points for each problem are in square brackets.

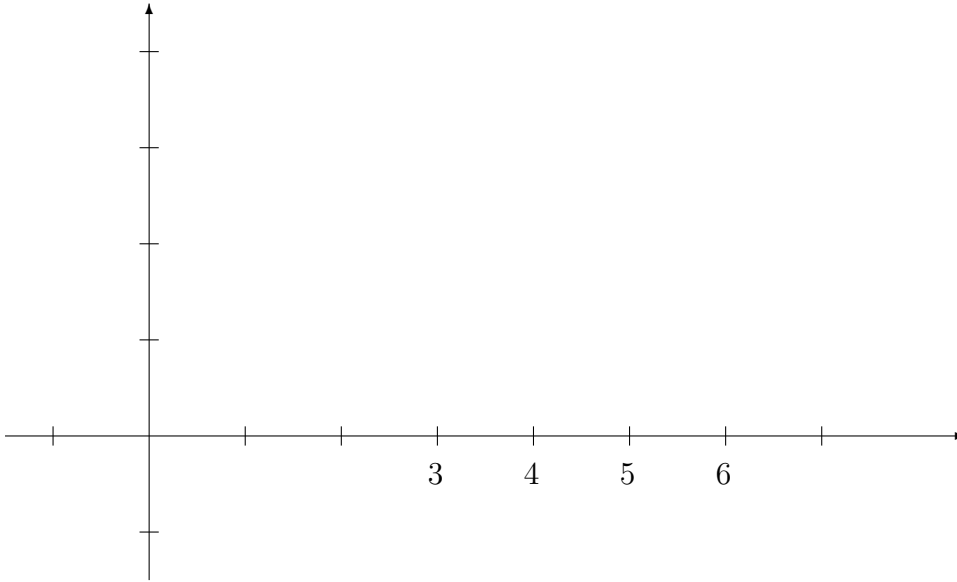
1. [16; 8 points each part] On limits of average rates of change. Let  $f(x) = \frac{1}{x}$ .

a. Write down an expression that gives the average rate of change of this function over the interval  $[x, x + h]$ , and simplify the expression.

b. Compute the limit as  $h \rightarrow 0$  of the average rate of change.

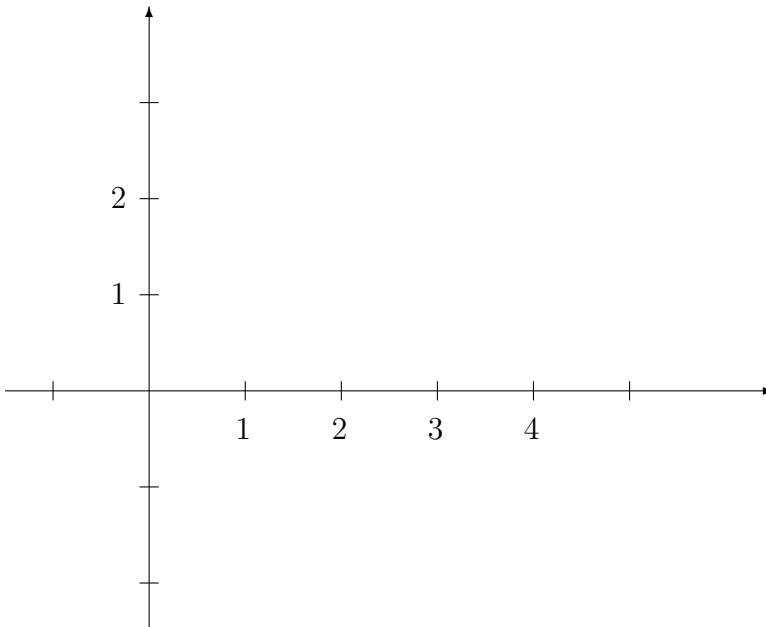
2. [15; 5 points for each property] On graphs, limits, and continuity. Draw the graph of *one* function  $f$  with domain  $(3, 6)$  that has all three of these properties:

- a.  $f$  is continuous on  $(3, 6)$  except at  $x = 4$  and  $x = 5$ , but  $f(4)$  and  $f(5)$  are defined.
- b. The limit  $\lim_{x \rightarrow 4} f(x)$  does not exist.
- c. The limit  $\lim_{x \rightarrow 5} f(x)$  does exist but  $f$  is not continuous at  $x = 5$ .



3. [15; 5 points each property] On asymptotes. Draw the graph of *one* function  $f$  that illustrates all three of these limits:

- a.  $\lim_{x \rightarrow 2^-} f(x) = \infty$ .
- b.  $\lim_{x \rightarrow 2^+} f(x) = -\infty$ .
- c.  $\lim_{x \rightarrow \infty} f(x) = 1$ .



4. [40; 8 points each part] Evaluate the following limits. If they diverge to  $\pm\infty$  it is enough to say they don't exist.

a.  $\lim_{x \rightarrow \pi} (x^2 + \sqrt{x} - 3 \cos x)$

b.  $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x^2 - 4}$

c.  $\lim_{x \rightarrow \infty} \frac{3x^3 - 2x^2 + x}{5x^2 + 4x + 1}$

d.  $\lim_{x \rightarrow 2} \frac{|2 - x|}{2 - x}$ .

e.  $\lim_{h \rightarrow 0} \frac{(x + h)^3 - x^3}{h}$

5. [16] Recall that we define  $\lim_{x \rightarrow a} f(x) = L$  to mean

$$\forall \epsilon > 0, \exists \delta > 0, \forall x (0 < |x - a| < \delta \Rightarrow |f(x) - L| < \epsilon).$$

Here, you will use that definition to prove that if  $\lim_{x \rightarrow 3} f(x) = 4$ , then  $\lim_{x \rightarrow 3} -f(x) = -4$ .

**a.** [4] Use the above definition of limits to translate what the given information,  $\lim_{x \rightarrow 3} f(x) = 4$ , says in terms of  $\epsilon$  and  $\delta$ .

**b.** [4] Translate what you're to prove,  $\lim_{x \rightarrow 3} -f(x) = -4$ , in terms of  $\epsilon$  and  $\delta$ .

**c.** [8] Now, let  $\epsilon > 0$  be given. Explain why there exists a  $\delta$  as required in part **b**. Point out where in your argument you use the information given in part **a**.

#1.[16]	
#2.[15]	
#3.[15]	
#4.[40]	
#5.[16]	
Total	