



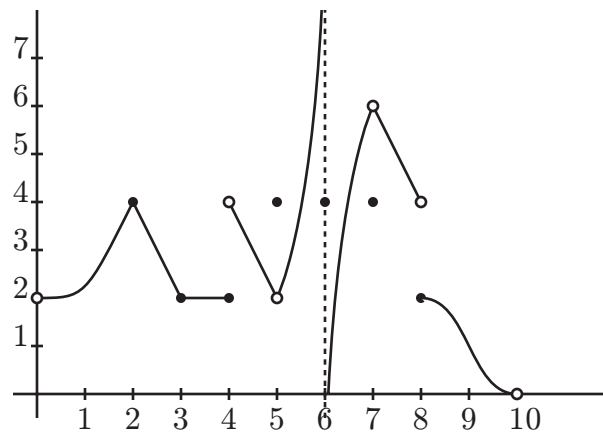
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Math 120 Calculus I
First Test Sample
October 2011

You may use a calculator. Leave your answers as expressions such as $e^2 \sqrt{\frac{\sin^2(\pi/6)}{1 + \ln 10}}$ if you like. Show all your work for credit. Points for each problem are in square brackets.

1. [18; 9 points each part] All parts refer to the graph of the function $f(x)$ with domain $0 < x < 10$ which is shown below. (All dots have integer coordinates and the dotted line is a vertical asymptote.)



- a. List the values of a in the interval $(0, 10)$ for which the limit $\lim_{x \rightarrow a} f(x)$ does not exist.
- b. List the values of a in the interval $(0, 10)$ at which f is not continuous.
2. [10] Let $f(x) = x^3$. Compute the average rate of change of f over the interval $[1, 3]$.
3. [20; 10 points each part] Recall that we define $\lim_{x \rightarrow a} f(x) = L$ to mean

$$\forall \epsilon > 0, \exists \delta > 0, \forall x (0 < |x - a| < \delta \Rightarrow |f(x) - L| < \epsilon).$$

Here, you will use that definition to prove that $\lim_{x \rightarrow 3} (10 - 2x) = 4$.

- a. Let $\epsilon > 0$. Determine what the value is needed for δ .
- b. In your own words, explain why whenever x is within your δ of 3, then $f(x)$ will be within ϵ of 4.

4. [42; 7 points each part] Evaluate the following limits. If they diverge to $\pm\infty$ it is enough to say they don't exist.

a. $\lim_{x \rightarrow 3} \frac{x^2 - 2x - 3}{x^2 - x - 6}$

b. $\lim_{x \rightarrow 3} \frac{2x^2 - 2x - 3}{x^2 - x - 6}$

c. $\lim_{x \rightarrow \infty} \frac{2x^2 - 2x - 3}{x^2 - x - 6}$

d. $\lim_{x \rightarrow 0} \frac{\cos x}{x + \sin x}$.

e. $\lim_{x \rightarrow 0} \frac{|x|}{x}$

f. $\lim_{x \rightarrow 0} \frac{3 \sin x}{2x}$

5. [10] Let $f(x) = 1 + x^2 - x^5$. Show that the equation $f(x) = -3$ has a solution in the interval $[0, 3]$. (Hint: IVT.)