



Name: _____

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Math 120 Calculus I
Second Test Sample
November 2011

You may use a calculator and one sheet of notes. Leave your answers as expressions such as $e^2 \sqrt{\frac{\sin^2(\pi/6)}{1 + \ln 10}}$ if you like. Show all your work for credit. Points for each problem are in square brackets.

(This sample test is a little longer than what the actual test will be.)

1. [15] Let g be the function inverse to the function $f(x) = x^3 + x$. Determine $g'(2)$. (Note that $g(2) = 1$.)
2. [15] Recall the following two limits we proved in class

$$\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1, \quad \text{and} \quad \lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\theta} = 0.$$

Use those two limits to prove that the derivative of $f(x) = \cos x$ is $-\sin x$. The sum formula for cosines will be useful here: $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$.

3. [40; 8 points each part] Differentiate the following functions. (Do not simplify your answers.)
 - a. $f(x) = \tan 3x + \arctan 3x$.
 - b. $f(x) = e^x \sqrt{x}$.
 - c. $f(x) = \frac{1 + \tan x}{x^2 + 5x + 2}$.
 - d. $f(x) = x^x$. (Suggestion: logarithmic differentiation)
 - e. $f(x) = \sqrt{\cos 2x}$.

4. [15] (Page 126, exercises 27-30.) Here are the graphs of four functions f_1 , f_2 , f_3 , and f_4 .

Match each of the following four graphs with their derivatives f'_1 , f'_2 , f'_3 , and f'_4 .

5. [15] Suppose that a drop of mist is a perfect sphere and that, through condensation, the drop picks up moisture at a rate proportional to its surface area $A = 4\pi r^2$. Show that under these circumstances the drop's radius r increases at a constant rate. (Recall that the volume V of a sphere is $V = \frac{4}{3}\pi r^3$.)