Math 120 Calculus I

Quiz Answers

December 2011

Scale. 8–9 A, 6–7 B, 4–5 C. Median 7.

For each of the following functions find all the critical points and determine if each gives a local maximum, local minimum, or neither.

1. [3] $f(x) = x^3 + x^2 - x$.

With all these you need to start by computing the derivative. For this one, the derivative is $f'(x) = 3x^2 + 2x - 1$. The critical points are where the derivative is 0, so next solve the equation $3x^2 + 2x - 1 = 0$.

The quadratic polynomial factors as $(3x - 1)(x + 1)$ so the solutions to the equation are $x = \frac{1}{3}, -1$. Those are the critical points.

There are various ways you can determine whether a critical point is a max or min. I'll use the second derivative test on this one. The second derivative is $f''(x) = 6x + 2$. Since $f''(\frac{1}{3})$ is positive, therefore $x = \frac{1}{3}$ gives a local max. And since $f''(-1)$ is negative, therefore $x = -1$ gives a local minimum.

2. [3] $f(x) = \dfrac{x}{1 + x^2}$.

Use the quotient rule $\left(\dfrac{u}{v}\right) = \dfrac{u'v - uv'}{v^2}$ to get $f'(x) = \frac{1(1+x^2) - x\,2x}{(1+x^2)^2} = \frac{1-x^2}{(1+x^2)^2}$.

The critical points occur when f' is 0, and that only happens when the numerator $1 - x^2$ is 0, that is, at $x = \pm 1$.

For this one I'll use the first derivative test since f'' will be messy to compute. There are only two places where f' is 0, namely $x = -1$ and $x = 1$, so on each of the three intervals $(-\infty, -1)$, $(-1, 1)$, and $(1, \infty)$ the derivative is positive or it is negative. On the first and last, it's negative, and on the middle interval it's positive. So the funtion f first decreases, then increases, then decreases, changing direction at the two critical points. Therefore, $x = -1$ gives a min, and $x = 1$ gives a max.

3. [4] $f(x) = x \ln x$.

Note that this function is defined only for positive values of x.

Use the product rule $u'v + uv'$ to get $f'(x) = 1 \ln x + x \frac{1}{x} = \ln x + 1$.

Solve the equation $\ln x + 1 = 1$ to find the critical points. $\ln x = -1$ when $x = e^{-1} = \frac{1}{e}$, and that's the only critical point.

Both the first and second derivative tests work quickly on this function. Here's the second derivative test: $f''(x) = \frac{1}{x}$. $f''(\frac{1}{e})$ is positive, so $x = \frac{1}{e}$ gives a minimum.