Syllabus
Math 120 Calculus I

D Joyce, Fall 2011

The text we'll use for the class is *University Calculus* by Hass, Weir and Thomas. We'll stress some topics and pass over some others.

Preview. Calculus is about the relation between a quantity and its rate of change. For an example, if the quantity is the distance travelled at a given time, then its rate of change is its velocity. If the velocity is constant, then calculus is not required: the distance travelled is the product of the elapsed time and the velocity. But when the velocity is not constant, then this formula doesn't apply. Nonetheless, the distance and velocity are intimately related. If the distance travelled at all times is known, then the velocity at any given time can be determined; and if the velocity at all times is known, then the distance travelled at any given time can be determined. These two operations are called *differentiation* and *integration*.

Much of calculus involves analyzing and developing these concepts and their applications.

Proofs are going to used throughout the course. When we first meet a concept, we'll discuss it intuitively. Then we'll formalize it with a formal definition. We'll use that definition to prove the things we expect to be true actually are true.

Review of functions. Chapter 1.

There are a slew of things in the first section of this chapter that I'll assume you know. We aren't going to cover them in class. I'll mention a couple of them, but chapter 1 is primarily there to show you some of the things that you're supposed to know already.

Besides what's in chapter 1, there are all the things on the page Mathematics background needed for calculus at `http://math.clarku.edu/~djoyce/ma120/background.html`

Limits and Continuity. Chapter 2.

After the introduction explaining where we're going, this is where the subject matter starts.

We first must clarify the concept of derivative. In some ways it is intuitively clear that a travelling body has a velocity, or more generally, any changing quantity has a rate of change. But just what is the rate of change? The answer is the rate of change at an instant is the limit of the average rates of change near that instant. That is, the derivative (instantaneous rate of change) of a function $f(x)$ is the limit of the average rate of change of the function over an interval $[x, x+h]$ as the length h of that interval approaches 0. The average rate of change over the interval is how much $f(x)$ changes over that interval divided by the length of the interval h. We want the limit of that average as the length h approaches 0.

The concept of limit is much more subtle than it first appears. We will discuss it in some detail and develop a formal definition of a limit and a formal notation to go along with it.

Key concepts associated to the concept of limit are tangent lines, limit laws, continuity, the sandwich theorem (also known as the pinching theorem), the formal definition for limits, discontinuities, asymptotes, continuity, the intermediate value theorem (IVT), and the definition of a derivative (which is why we study limits in the first place).

Chapter 2. Limits and Continuity
§2.1. Rates of change and tangents to curves
§2.2. Limit of a function and limit laws
§2.3. The precise definition of a limite
§2.4. One-sided limits and limits at infinity
§2.5. Infinte limits and vertical asymptotes
§2.6. Continuity
§2.7. Tangents and derivatives at a point

Chapter 3. Derivatives. Now that we've got a solid definition of limit (that was section 2.7), we can start to study derivatives. There are a number of rules for differentiation (finding derivatives), mostly easily learned, although the chain rule, for some reason, seems to be more difficult to master. There are a couple of different notations for derivatives that everyone uses.

I assume that you know the trig functions, sine, cosine, etc., and we will find and use their derivatives. Further topics in differentiation include higher derivatives, implicit differentiation, differentiation of inverse functions, and differentiation of logarithms. We'll finish this chapter studying situations when several quantities are changing; we'll see how their derivatives are related.

Chapter 3. Differentiation
§3.1. The derivative of a function
§3.2. Differentiation rules for polynomials, exponentials, products, and quotients
§3.3. The derivative as a rate of change
§3.4. Derivatives of trigonometric functions
§3.5. The chain rule and parametric equations
§3.6. Implicit differentiation
§3.7. Derivatives of inverse functions and logarithms
§3.8. Inverse trigonometric functions
§3.9. Related rates

Applications of Derivatives. Chapter 4.

The applications of derivatives are numerous. Besides classical applications in physics and the natural sciences, there are applications in the social sciences, for instance, marginal profits are just derivatives of profits.

We'll mix theory with applications by proving the methods work. We'll prove and use the Mean Value Theorem (MVT) so show our expectations are correct. For instance, if the derivative is positive, then the function is increasing; at a maximum or a minimum of a function, the derivative is zero. We'll do some curve sketching as a way to get a better understanding of the relation between a function and its derivative. We'll also see what second derivatives mean for a function and the graph of the function.

Chapter 4. Applications of derivatives
§4.1. Extreme values of functions
§4.2. The Mean Value Theorem (MVT)
§4.3. Monotonic functions and the first derivative test
§4.4. Concavity and curve sketching

§4.5. Applied optimization
§4.6. Indeterminant forms and L'ôpital's rule

Math 120 Home Page at `http://math.clarku.edu/~djoyce/ma120/`