Name: _____

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Math 122 Calculus III

First Test, October 2011

You may use a calculator. Leave your answers as expressions such as $e^2\sqrt{\dfrac{\sin^2(\pi/6)}{1+\ln 10}}$ if you like. Show all your work for credit. Points for each problem are in square brackets.

Problem 1. On improper integrals. [30; 15 points each part]

For each improper integral, determine whether or not it converges. If it does converge, give its value.

a. $\displaystyle\int_2^\infty \frac{2}{x^2-1}\,dx$. (Note: $\dfrac{2}{x^2-1} = \dfrac{1}{x-1} - \dfrac{1}{x+1}$.)

b. $\displaystyle\int_2^\infty \frac{1}{x\ln x}\,dx$. (Suggestion: substitution.)

Problem 2. On limits. [32; 8 points each] Determine the following limits. If you use l'Hôpital's rule, point out where you use it.

a. $\lim_{x \to 1} \dfrac{\ln x}{1 - x}$

b. $\lim_{x \to 0} \left(\dfrac{1}{\ln(1 + x)} - \dfrac{1}{x} \right)$

c. $\lim_{x \to 0} x^x$

d. $\lim_{n \to \infty} \dfrac{2^n + 5^n}{3^n + 5^n}$

Problem 3. On proofs of convergence. [20]

Recall that a sequence is said to *converge* to the *limit* L, if for each $\epsilon > 0$, there is some N such that beyond the N^{th} term, every term is within ϵ of L. Symbolically, this convergence says

$$\forall \epsilon > 0, \exists N, \forall n \geq N, |a_n - L| < \epsilon.$$

Consider the sequence whose n^{th} term is $a_n = 1/2^n$. Its limit L is 0. Let ϵ be an arbitrary positive number. Find a value of N so that for $n \geq N$ it is the case that $|a_n - L| < \epsilon$. (Note: your answer should express N in terms of ϵ.)

Problem 4. [20; 4 points each part] True/false. For each sentence write the whole word "true" or the whole word "false". If it's not clear whether it should be considered true or false, you may explain in a sentence if you prefer.

_____ **a.** If the n^{th} term of a sequence a_n lies between $2 \pm 1/n$, then $\lim_{n\to\infty} a_n = 2$.

_____ **b.** If $a_n \to 0$ and $b_n \to 1$, then $a_n^{b_n} \to 0$.

_____ **c.** If the terms of a sequence are decreasing and bounded above, then the sequence converges.

_____ **d.** If $a_n \to \pi/2$, then $\tan a_n$ is a divergent sequence.

_____ **e.** If $\dfrac{a_n}{b_n} \to 3$ and $b_n \to 0$, then $a_n \to 0$.

#1.[30]	
#2.[32]	
#3.[20]	
#4.[20]	
Total	