First test extra credit

Math 122 Calculus III
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Extra credit problems. If you didn't do as well as you thought you could and would like to show what you really know, you can do these extra credit problems. If you do them all and do them well, I'll add back half the points you missed on your first test score.

You may use notes, textbooks, and a calculator, but don't talk to anyone when doing the extra credit problems. If you have any questions, ask me or email me your questions. Finish the extra credit problems by Friday, Oct 21.

Problem 1. On improper integrals.

a. In your own words, describe what it means for an integral to be an improper integral and what it means for an improper integral to converge or diverge.

b. Give an example of an improper integral that diverges and show it diverges.

c. Also give an example of an improper integral that converges and show what it converges to.

Problem 2. On indeterminant forms.

a. In your own words, explain what it means when someone says $\frac{0}{0}$ is an indeterminant form.

b. List 7 indeterminant forms. Each should be a quotient, product, difference, or power where the operands are chosen from 0, 1, and ∞.

c. Give an example of a limit of the form $\frac{\infty}{\infty}$ which diverges.

d. Give an example of a limit of the form $0 \times \infty$ which converges and show it converges.

Problem 3. On the concept of convergence of sequences.

Consider the sequence whose n^{th} term is $a_n = \dfrac{1}{\ln n}$. Its limit L is 0. Let ϵ be an arbitrary positive number. Find a value of N so that for $n \geq N$ it is the case that $|a_n - L| < \epsilon$. (Note: your answer should express N in terms of ϵ.)