Analysis of the harmonic series and Euler's constant
Math 122 Calculus III
D Joyce, Fall 2012

Consider the standard harmonic series

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} + \cdots$$

1. We briefly discussed the divergence of this series in class using Oresme's argument where he grouped terms together. Explain that proof in your own words in such a way that an elementary school student could understand your argument.

2. Read up on Nicole Oresme (1320–1382). You'll find various summaries on the web. Try to find where it was that he actually gave this proof. You may have to use sources in our library.

3. As usual, let S_n be the n^{th} partial sum of this series. $S_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$. Fill in the following table. A calculator will help. If you like, you can write a computer program to do the work, and if you do, then fill in lines for $n = 100, 1000, 10000, 10000, 100000$, and 1000000 as well.

n	S_n	$\ln n$	$S_n - \ln n$
1	1.0000	0.0000	1.0000
2	1.5000	0.6931	0.8069
3	1.8333	1.0986	0.7347
4			
5			
10			
20			

4. Use the integral test to explain why S_n should be close to $\ln n$. Illustrate your explanation with figures. Show how to interpret the difference $S_n - \ln n$ in your figure.

5. Argue that $\lim_{n\to\infty} (S_n - \ln n)$ exists.

6. Denote that limit by the Greek letter gamma, γ. This is called *Euler's constant*. Read up on Leonhard Euler (1707–1783) and Euler's constant, and find one other property of γ (besides its definition) and describe that property.

Math 122 Home Page at `http://math.clarku.edu/~djoyce/ma122/`