Summary: polar and parametric
Math 122 Calculus III
D Joyce, Fall 2012

This is a summary sheet about the topics we've discussed in polar coordinates and parametric equations.

Polar coordinates and complex numbers A point in the plane is described in rectangular coordinates by a pair (x, y) of real numbers. It can be identified with the complex number $x + yi$.

It can also be described in polar coordinates by the pair $[r, \theta]$ where

$$\begin{aligned} x &= r\cos\theta & r^2 &= x^2 + y^2 \\ y &= r\sin\theta & \tan\theta &= y/x \end{aligned}$$

As complex numbers, $x + yi = r(\cos\theta + i\sin\theta)$.

Curves in the plane can be described either in rectangular coordinates as an equation in x and y, or in polar coordinates as an equation in r and θ.

When an curve is given in polar coordinates as a function $r = f(\theta)$, we can find the area inside the curve and between two rays $\theta = \alpha$ and $\theta = \beta$ as the integral $\displaystyle\int_\alpha^\beta \tfrac{1}{2}r^2\,d\theta$.

The slope at a point of a curve in polar coordinates is $\displaystyle\frac{dy}{dx} = \frac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta}$.

Parametric equations The path of a point at time t is given by a pair of equations $x = x(t)$ and $y = y(t)$. In that case we say that the curve is described by the parameter t. The position of the point at time t is the vector (x, y). Any variable can be used as a parameter, but usually the parameter is t or θ.

The velocity of the point at time t is $(x', y') = (x'(t), y'(t))$, and its acceleration is $(x'', y'') = (x''(t), y''(t))$. It's speed is $s = \sqrt{x'^2 + y'^2}$.

The slope at the point (x, y) of the curve is $\displaystyle\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$.

The area under a curve given parametrically for $a \le t \le b$ is $\displaystyle\int_a^b = yx'\,dt$.

The length of the curve for $a \le t \le b$ is $\displaystyle\int_a^b ds = \int_a^b s\,dt = \int_a^b \sqrt{x'^2 + y'^2}\,dt$.

Math 122 Home Page at `http://math.clarku.edu/~djoyce/ma122/`