Math 126, first test sample problems.

These are sample questions. You may bring one sheet of prepared notes for the test. Calculators are optional; you may bring one if you like.

Problem 1. Carefully prove the following statement. If p is a prime, and n any positive integer, then the greatest common divisor (p, n) is either 1 or p.

Problem 2. On the Euclidean algorithm.

a. Apply the Euclidean algorithm to show that the greatest common divisor of 1001 and 805 is 7.

b. Use the results of the computation in part a to express 7 as a linear combination of 1004 and 805.

c. For each of the following two linear Diophantine equations, either find a solution, or explain why no solution exists.

$$1001x + 805y = 14$$
$$1001x + 805y = 3$$

Problem 3. On divisors.

a. Draw the Hasse diagram of the divisors of 200.

b. What are the values of $d(200)$ and $\sigma(200)$.

c. Is the number 200 a perfect number? Why or why not?

Problem 4. Since 19 is a prime number, $\mathbf{Z}_{19}$ is a field, and division, except by 0, works in $\mathbf{Z}_{19}$. Thus, there is some x such that

$$6x \equiv 1 \pmod{19}.$$

Find such an x. In one sentence, explain the method you used to find the solution.

Problem 5. Prove that if $a \equiv b \pmod{n}$, and $c \equiv d \pmod{n}$, then $a - c \equiv b - d \pmod{n}$.

Problem 6. True or false. Just write the word "true" or the word "false". If it's not clear to you which it is, explain; otherwise no explanation is necessary.

a. The principle of mathematical induction is used to make conjectures about numbers, but it sometimes makes wrong conclusions.

b. If a, b, and c are integers, then $(a, b) = (a - cb, b)$.

c. If an integer n is not a perfect cube (i.e., not the cube of any integer), then the cube root $\sqrt[3]{n}$ is an irrational number.

d. Although Euclid devoted three of the books of his *Elements* to number theory, he stated no axioms for numbers.

e. A Pythagorean triple consists of three positive integers a, b, and c such that $a^2+b^2 = c^2$.