Math 126, Number Theory

Sample questions for the second test

April 2006

The test will cover sections 3.4, 3.5, 3.7, and chapter 5. You may bring one sheet of prepared notes for the test. Calculators are optional; you may bring one if you like.

Proofs. There will be a couple short proofs on the test.

1. Show that the number $341 = 11 \cdot 31$ is a pseudoprime. Recall that the definition that we're using for pseudoprime is that n is a pseudoprime if $2^n \equiv 2 \pmod{n}$. (Hints: show that 2^{341} is congruent 2 both modulo 11 and modulo 31, and explain why that implies $2^{341} \equiv 2 \pmod 341$.)

2. Show that the Diophantine equation $x^2 - 3y^2 = 2$ has no solutions. (Hint: modulo 3.)

3. Show that if $n > 2$ then $2|\phi(n)$.

Computations. There will be a couple of computational problems on the test.

4. The prime factorization of 105875 is $5^3 \cdot 7 \cdot 11^2$. Compute $\phi(105875)$. You may leave your answer as an arithmetical expression or you may evaluate it as a number.

5. Determine how many primitive roots there are modulo 101. (Note 101 is prime.)

6. Find all solutions in positive integers to $x^2 + 12 = y^4$.

True/False questions. There will be some of these on the test. Here's a sample of true/false questions.

7. 2 is a primitive root modulo 7 since $2^6 \equiv 1 \pmod 7$.

8. The equation $x^2 - 17y^2 = 3$ is an example of a Pell equation (also called a Fermat/Pell equation).

9. The number 4926834923 is not the sum of two squares since any square modulo 4 is congruent to 0 or 1.

Short answers. The rest of the test will consist of questions that you can answer in a sentence or two.

10. We've worked out the solutions to several higher-degree Diophantine equations by rewriting them in a form like $x^n = yz$ where n was 2 or 3. We then somehow took care of the cases where y and z weren't relatively prime, and that left us with the case where y and z are relatively prime. Explain why the case were y and z are relatively prime is easily solved.

11. Explain why the solutions to the Diophantine equation $x^2 + y^2 = z^2$ are geometrically significant.

12. Describe Fermat's method of descent.