Math 126 Number Theory

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Due Wednesday. From page 14, Misc. exercises: 1, 2, 3, 4.

Last meeting. Introduction to the course including a brief survey of number theory. We also talked about prime numbers, divisibility of numbers, and Pythagorean triples.

Definition. An integer m *divides* an integer n, written $m|n$, if there exists an integer k such that $mk = n$. When m does not divide n, we denote that as $m \nmid n$.

Definition. An integer p greater than 1 is said to be *prime* if it's only positive divisors are 1 and p, otherwise it is said to be *composite*.

We looked at this definition Euclid's *Elements*. Euclid of Alexandria lived about 300 B.C.E.

We noted that in order to determine whether a number n was prime, it was enough to see if any prime p less than or equal to $\sqrt{n}$ divides n. If not, then n is prime. But we didn't find the reason why that's enough.

Definition. A Pythagorean triple $x : y : z$ of three positive integers x, y, and z, satisfies the equation $x^2 + y^2 = z^2$.

These were also mentioned in the *Elements*. Equations like that where the solutions are required to be integers or rational numbers are called *Diophantine equations* in honor of Diophantus of Alexandra (about 500 to 600 years after Euclid) who studied them.

We looked at a couple of them, namely $3 : 4 : 5$ and $5 : 12 : 13$, noted that a multiple $nx : ny : nz$ of a Pythagorean triple was another Pythagorean triple, and discovered another one with the help of a little algebra. We'll look at Pythagorean triples in more detail later in the course.

We talked about a couple of things Fermat (1601–1665) did. One was his claim that all numbers of the form $F_n = 2^{2^n} + 1$ were prime. He knew that this was true for $n = 0, 1, 2, 3,$ and 4, but it turns out that it's not true for $n = 5$.

Fermat also wrote in his book of Diophantus that he had a proof that there were no positive integers x, y, and z that satisfied any equation $x^n + y^n = z^n$ for any $n \geq 3$. That was finally proved by Andrew Wiles about ten years ago.

Today. We'll look at a couple more things in chapter 1, and we may start a discussion of even and odd numbers.