Math 126 Number Theory

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Due Today. Page 155: exercises 1, 2, 7. Choose one of the three and write it up completely.

Due Monday. Page 161: exercises 3, 5. Choose one and write it up completely. Same instructions as for Friday's assignment.

Second test. Wednesday, April 5.

Last time. We discussed decimal representations of rational numbers. some more. Here are a few notes from our discussion.

Nonterminating decimal expansions. Decimal fractions have infinitely many nonzero digits. Even those that terminate, such as $\frac{1}{8} = 0.125$ can be considered as having infinitely many digits, the rest all being 0. $\frac{1}{8} = 0.12500000....$

You probably know that they all are repeating, or at least eventually repeating. For example,

$$\frac{1}{7} = .142857142857142857142857...$$

has a decimal expansion that consists of the six digits 142857 over and over, while

$$\frac{83}{74} = 1.1216216216216216216...$$

has a decimal expansion that starts 1.1 then has the three digits 216 repeated over and over.

We'll use a standard notation to indicate the repeating part of a decimal expansion, and that's to put a line over the repeating part. Thus, $\frac{1}{7} = .\overline{142857}$ and $\frac{83}{74} = 1.1\overline{216}$.

Our first question was: how do you compute decimal expansions? Answer: long division.

A more difficult question was: why do all these decimal expansions of rational numbers eventually repeat? We answered that by looking at the remainders in the process of long division. Since all the remainders are between 0 and $n-1$, there are only n different possible remainders. So, by the $n+1^{\text{st}}$ remainder we've got a repetition. If two remainders are the same, then the rest of the long division process repeats, in particular, the quotient digits repeat.

Today. We'll look over a some of your answers of the exercises due today. Time permitting, we'll discuss decimal representations of rational numbers some more.

More on decimal expansions. Another question, which you probably know the answer to, is: what rational numbers $\frac{m}{n}$ have finite decimal expansions? We can assume that $\frac{m}{n}$ is already in reduced form, that is $(m, n) = 1$. After we formulate the answer, we'll prove it.

If we have time we'll look into the period for the decimal expansion of $\frac{m}{n}$.