

Math 128, first test sample problems.

These are sample questions. You may bring one sheet of prepared notes for the test. Calculators are optional; you may bring one if you like.

Problem 1. On complex numbers.

- Write the reciprocal of $5 + 12i$ in the form $a + bi$.
- What are the two square roots of i ?
- Write $e^{i\pi/3}$ in the form $a + bi$.
- Write $5 + 5i$ in polar form $re^{i\theta}$.

Problem 2. On transformations.

- Let S and T be two translations on the complex plane \mathbf{C} . Prove that their composition $T \circ S$ is also a translation.
- The transformation $T(z) = iz + (1 - i)$ is a rotation. Determine (1) its fixed point, and (2) its angle of rotation.
- Give a formula for the transformation T of the complex plane \mathbf{C} which is a scaling (that is, a homothetic transformation) that fixes 0 and sends $3i$ to $4i$.

Problem 3. On the stereographic projection. Recall that with the stereographic projection the point (a, b, c) on the unit sphere $S^2 = \{(x, y, z) \mid x^2 + y^2 + z^2 = 1\}$ corresponds to the point $z = \frac{a + ib}{1 - c}$ in \mathbf{C}^+ . The only exception to this correspondence is that the “North pole” $(0, 0, 1)$ of the sphere S^2 corresponds to the point ∞ in \mathbf{C}^+ .

The plane $x + y + z = 1$ in space \mathbf{R}^3 intersects the unit sphere S^2 in a circle. By means of the stereographic projection, that circle corresponds to some curve in \mathbf{C}^+ . Describe either in words or by means of an equation what curve that is. Explain your answer.

Problem 4. On transformation groups. Consider the collection G of transformations of the plane \mathbf{C} that includes all translations and all half turns.

- In order for G to be a transformation group, three conditions have to be satisfied. State each condition, and after you state it, explain why G satisfies that condition.
- Let ABC be an equilateral triangle. Describe an equilateral triangle DEF whose sides are equal to those of ABC , but, with respect to the group G , triangle DEF is *not* congruent to triangle ABC .

Problem 5. On Möbius transformations.

- There is one Möbius transformation T that maps $0 \mapsto 1$, $\infty \mapsto 0$, and $1 \mapsto \infty$. What is it?
- Determine the fixed points of the transformation T you found in part a.

Problem 6. Prove that the only Möbius transformations that have a single fixed point at ∞ are translations.