CLARK

Name: ____________________

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Math 130 Linear Algebra

Final

Dec 2006

You may refer to one sheet of notes on this test, and you may use a calculator. You may leave your answers as expressions such as $\binom{8}{4}\frac{e^{1/3}}{\sqrt{2\pi}}$ if you like. Points for each problem are in square brackets.

Problem 1. [14; 7 points each part] Let A be the matrix $A = \begin{bmatrix} 1 & 5 & 0 & 2 & -2 \\ 0 & 1 & 0 & 3 & 4 \\ 0 & 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 2 & 14 \end{bmatrix}$.

a. Recall that the row space of an $m \times n$ matrix is the subspace of $\mathbf{R}^n$ spanned by the rows of the matrix. Find a basis for the row space for A.

b. Recall that the null space of a matrix A is the set of all solutions of the homogeneous system $A\mathbf{x} = \mathbf{0}$. Find a basis for the null space for A.

Problem 2. [14; 7 points each part] Consider the three vectors $\mathbf{u} = (1, 3, 1)$, $\mathbf{v} = (4, 2, -1)$, and $\mathbf{w} = (-3, 1, 2)$.

a. Either prove that the $\mathbf{u}$ is in the span of the vectors $\mathbf{v}$ and $\mathbf{w}$, or prove that it is not.

b. Are the three vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ linearly dependent, or linearly independent?

Problem 3. [10] A parallelogram in $\mathbf{R}^3$ has as adjacent sides the vectors $\mathbf{u} = ((1, 3, -2)$ and $\mathbf{v} = (3, -1, -1)$. Determine the area of the parallelgram.

Problem 4. [20; 5 points each part] Let A be the matrix $A = \begin{bmatrix} 2 & 0 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$.

a. Write down the characteristic polynomial $f(\lambda)$ for A.

b. Determine the eigenvalues for A.

c. For each of the eigenvalues of A, find the eigenspace of eigenvectors for that eigenvalue.

d. Is A a diagonalizable matrix? Explain why or why not.

Problem 5. [20; 10 points each part] Recall that a subset W of a vector space V is a subspace of V if and only if (1) $\mathbf{0}$ is a vector in W, (2) W is closed under vector addition, and (3) W is closed under scalar multiplication.

a. Prove that the intersection

$$W_1 \cap W_2 = \{\mathbf{v} \in V \mid \mathbf{v} \in W_1 \text{ and } \mathbf{v} \in W_2\}$$

of any two subspaces W_1 and W_2 of a vector space V is also a subspace of V.

b. Give an example that shows that the union of two subspaces does not have to be a subspace. For your example, specify what the vector space V is, what the two subspaces W_1 and W_2 of V are, and explain why the union

$$W_1 \cup W_2 = \{\mathbf{v} \in V \mid \mathbf{v} \in W_1 \text{ or } \mathbf{v} \in W_2\}$$

is not another subspace of V.

Problem 6. [12; 4 points each part] On dimension and basis. Let V be the vector space $V = \{(w, x, y, z) \in \mathbf{R}^4 \mid w = x + y + z\}$.

a. What is the dimension of V? Explain how you know that dimension.

b. Exhibit a basis for V. (No need to explain how you found it.)

c. Give an example of a 2-dimensional subspace W of V.

Problem 7. [10] If A is a 5×3 matrix, show that the rows of A are linearly dependent.