Math 130 Linear Algebra

Prof. D. Joyce, Clark University

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Due today. More exercises from section 4.2: 25, 26, 27ab, 31, 32, 34, T7, T10.

Quiz Friday on sections 4.1 and 4.2.

Due Monday. Exercises from section 4.3: 1, 4–7, 21, 22.

Due next Wednesday. More exercises from section 4.3: 25, 26, 27, 29, T9, T10, T11.

Last time. Unit vectors in n-space, standard unit vectors, and linear transformations $L : \mathbf{R}^n \to \mathbf{R}^m$.

For next time read section 5.1 on cross products in $\mathbf{R}^3$.

Today. Finish notes from last time, that part that shows each linear transformation $L : \mathbf{R}^n \to \mathbf{R}^m$ is given by a unique $m \times n$ matrix A, called the standard matrix representing L. Discuss graph theory, section 2.2.

Directed graphs. The graphs we're talking about aren't the graphs of functions like you see in calculus, but graphs that are studied in combinatorics and graph theory.

A *graph* G consists of *vertices* (also called points or nodes) and *edges* (also called lines or arrows). Each edge *joins* two vertices. Usually we'll assume that at most one edge connects any pair of vertices, when more are allowed, it's called a *multigraph*. When the order that they join the vertices is noted, then the graph is called a *directed graph* or *digraph*, when that order is not noted, it's called an *undirected graph*.

The graphs we'll consider are finite graphs where there are only finitely many vertices and edges.

One reason directed graphs occur so often is that they're just binary relations on a finite set, where the vertices are taken to be the set, and an edge is connected between two elements of the set if the relation holds between the to elements. Example: Set of people in a town. The relation holds between P and Q if P is a parent of Q.

Graphs are used in both natural and social science. Our text discusses influence graphs as an example from social science. Many applications have more structure than just a graph. Sometimes there's information attached to the vertices, sometimes to the edges, sometimes to both. For flight paths of airlines, the vertices could be labelled with the airfield name and the edges with information like the distance between fields (giving an undirected graph); or, perhaps, each flight would give an edge with the time that the flight occurs (giving a directed multigraph).

A directed graph G can be described by an *adjacency matrix* $A = A(G)$. Label the vertices 1 through n, where n is the number of vertices. Let the entry in the ith row and j^{th} column of the adjacency matrix A be 1 if some edge connects them, but 0 if not.

If you've got an undirected graph, then the ad-

jacency matrix is symmetric.

One of the natural questions that comes up when you're studying graphs is whether or not there is a path from one vertex to another, or, more generally, how many paths there are. The adjacency matrix A itself tells you whether there are paths of length 1 between vertices. Its square, A^2, gives you the number of paths of length exactly 2 between vertices; likewise A^n, for those of length exactly n. Instead of taking the nth power of A you take the nth power of $I + A$, you get paths of lengths less than or equal to n (where every vertex is connected to itself by a path of length 0). If you don't want to consider paths of length 0, then $A + A^2 + \cdots + A^n$ gives you the number of paths of length from 1 through n. (Note: if $I - A$ is an invertible matrix then $A + A^2 + \cdots + A^n = A(I - A^n)(I - A)^{-1}$ just as you would expect.)