Math 131 Multivariate Calculus
Diagnostic Test Answers

Results. Problem 1 was done well by everyone. We'll need to do some review of parametric equations like in problem 2. Problem 3 was pretty well done, but we'll need to review how the angle between two vectors is related to the their dot product. We'll need to review cross products and their interpretation like in problem 4.

Problem 1. On vectors and vector arithmetic.

a. Add these vectors. $(3, 2, -4) + (2, -1, 0)$

Just add the coordinates. $(5, 1, -4)$.

b. Compute $5(1, 2) - 3(2, 2) + (0, 3)$.

That equals $(5, 10) - (6, 6) + (0, 3) = (-1, 7)$.

c. The displacement vector from $(0, 1, 2)$ to $(3, 3, 5)$ is

Take the difference. The displacement vector from P to Q is $Q - P$, which in this case equals $(3, 2, 3)$.

d. If the vector $\mathbf{a}$ goes from point P to point Q while the vector $\mathbf{b}$ goes from point P to point R, then the vector $\mathbf{b}-\mathbf{a}$ goes from which point to which point?

From Q to R. You can actually think of $\mathbf{a}$ as being $Q-P$, and $\mathbf{b}$ as $R-P$. Then $\mathbf{b}-\mathbf{a} = (R-P)-(Q-P) = R-Q$.

e. Write the vector $4\mathbf{i} + 5\mathbf{j} - 6\mathbf{k}$ in coordinate form.

It's $(4, 5, -6)$.

Problem 2. On parametric curves.

a. The curve given in parametric form by the equations $(x, y) = (\cos\theta, \sin\theta)$ is what?

A circle. In fact, it's the unit circle.

b. Give parametric equations $(x(t), y(t), z(t))$ with parameter t for the straight line passing through the points $(1, 2, 1)$ and $(3, 6, -1)$.

Here's one solution:

$$(x, y, z) = (1, 2, 1) - t\,((3, 6, -1) - (1, 2, 1))$$

so $x(t) = 1 + 2t$, $y(t) = 2 + 4t$, and $z(t) = 1 - 2t$.

Problem 3. On dot products and lengths of vectors.

a. The length (also called norm) of the vector $(0, 4, 3)$ is what?

In general, the length of a vector (a, b, c) is $\sqrt{a^2 + b^2 + c^2}$, so the length of this vector is $\sqrt{4^2 + 3^2}$ which equals $\sqrt{25} = 5$.

b. Compute the dot product (also called inner product or scalar product) $(2, 3, 1) \cdot (2, -1, 2)$.

The dot product of two vectors is found by summing products of corresponding coordinates. Here it equals $2 \cdot 2 + 3 \cdot (-1) + 1 \cdot 2 = 4 - 3 + 2 = 3$.

c. A unit vector in the direction of $(1, 1)$ is what?

A unit vector is a vector of length 1. To find one in the direction of a vector $\mathbf{v}$, just divide $\mathbf{v}$ by its length. Since the length of $(1, 1)$ is $\sqrt{2}$, therefore a unit vector in that direction is $(1, 1)/\sqrt{2}$ which you can also write as $\left(\frac{1}{2}\sqrt{2}, \frac{1}{2}\sqrt{2}\right)$.

d. The distance between the vectors $(2, 2)$ and $(7, 14)$ is what?

It's the length of the difference $(5, 12)$, which is $\sqrt{169} = 13$.

e. If $\mathbf{a} = (2, 4)$ and $\mathbf{b} = (1, 3)$, then the cosine of the angle θ between $\mathbf{a}$ and $\mathbf{b}$ is what?

$$\cos\theta = \frac{\mathbf{a}\cdot\mathbf{b}}{\|\mathbf{a}\|\,\|\mathbf{b}\|} = \frac{14}{\sqrt{20}\,\sqrt{10}}$$

which you can simplify if you want to.

Problem 4. On determinants, cross products, and areas.

a. Evaluate the determinant

$$\begin{vmatrix} 2 & 3 & 1 \\ 0 & -1 & 2 \\ 1 & 0 & 4 \end{vmatrix} = -8 + 6 + 1 = -1$$

b. Compute the cross product (also called outer product or vector product)

$$(2, 0, 1) \times (0, 1, 2) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 0 & 1 \\ 0 & 1 & 2 \end{vmatrix}$$

which equals $-\mathbf{i} - 4\mathbf{j} + 2\mathbf{k} = (-1, -4, 2)$.

c. Name a vector perpendicular to the vectors $(2, 0, 1)$ and $(0, 1, 2)$ used in part **b**.

The cross product of two vectors is perpendicular to both, so $(-1, -4, 2)$ works.

d. Consider the parallelogram in $\mathbf{R}^3$ three of whose vertices are $(0, 0, 0)$ and the two vectors $(2, 0, 1)$ and $(0, 1, 2)$ used in part **b**. The area of this parallelgram is what?

It's the length of the cross product, namely $\sqrt{1 + 16 + 4} = \sqrt{21}$.