Math 131 Multivariate Calculus

Quiz Answers

5 Feb 2010

Scale. 5 points each problem. 8–10 A, 6–7.5 B.

1. Evaluate this limit, or explain why it doesn't exist. (You may do your computations either in rectangular coordinates as written, or convert to polar coordinates.)

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$$

First note that both the numerator and denominator approach 0, so more analysis is needed. You probably checked to see what the limits are along the x- and y-axes. Along the x-axis, where $y=0$, we have $\lim_{x\to 0} \frac{0}{x^2} = 0$, and along the y-axis, where $x=0$, we get the same limit, 0. So, it looks like the limit might be 0.

But for the limit to be 0, it has to be 0 along every line approaching 0. Along the line $y=x$ we get a different limit:

$$\frac{xy}{x^2+y^2} = \frac{x^2}{2x^2} \to \frac{1}{2}.$$

Therefore, the vector limit doesn't exist. (In fact, if you approach $(0,0)$ along any line except the two axes, you'll get a nonzero limit.)

Here's how you could conclude the limit didn't exist by using polar coordinates. Use the equations $x = r\cos\theta$, $y = r\sin\theta$, and $x^2+y^2=r^2$. For the limit to exist, for all θ as $r\to 0$, the limit must have the same value, but

$$\lim_{r\to 0} \frac{r^2\cos\theta\sin\theta}{r^2} = \cos\theta\sin\theta$$

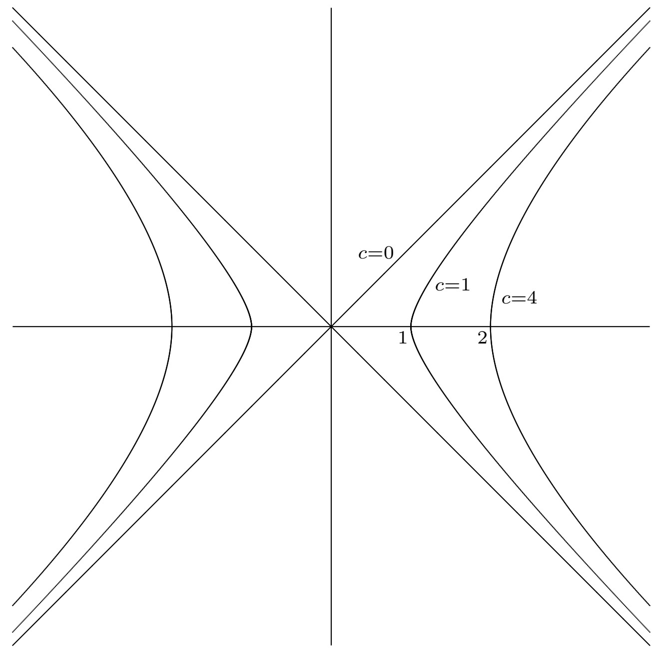
has different limits for different values of θ. Therefore the vector limit doesn't exist.

Problem 2. Consider the function $f(x,y) = x^2 - y^2$. Graph in the (x,y)-plane the level curves for the following three heights c for this function. (Note that since all the equations are quadratic, they're all conic sections—ellipses, parabolas, hyperbolas, or degenerate forms of those.)

a. $c=1$. **b.** $c=4$. **c.** $c=0$.

For $c=1$ the level curve is the graph of the equation $x^2-y^2=1$. That's a hyperbola with asympototes $y=\pm x$. It passes through the points $(\pm 1, 0)$.

For $c=4$ the level curve is the graph of the equation $x^2-y^2=4$. That's a hyperbola with asympototes $y=\pm x$. It passes through the points $(\pm 2, 0)$.



For $c=0$ the equation is $x^2-y^2=0$, equivalently $y^2=x^2$, so $y=\pm x$. The level curve is the union of the two lines $y=\pm x$, which are the asympototes of the two parabolas above.

The graph of this function, $z = x^2-y^2$ is a called a *hyperbolic paraboloid*. We've seen some of its planar cross sections are hyperbolas. If x or y is held constant, the planar cross sections you get are parabolas.