Math 131 Multivariate Calculus

Quiz on Dot and Cross Products

February 2014

Scale. 10–12 A, 8–9 B, 5–7 C. Median 8.5.

1. [4; 2 points each part] Name a vector perpendicular to the vectors $\mathbf{a} = (2, 0, 1)$ and $\mathbf{b} = (0, 1, 2)$

a. Name a vector that is perpendicular to both **a** and **b**.

It's the cross product

$$(2,0,1) \times (0,1,2) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 0 & 1 \\ 0 & 1 & 2 \end{vmatrix}$$

which equals $-\mathbf{i} - 4\mathbf{j} + 2\mathbf{k} = (-1, -4, 2)$.

b. Consider the parallelogram two of whose sides are **a** and **b**. The area of this parallelgram is

It's the length of the cross product, namely $\sqrt{1 + 16 + 4} = \sqrt{21}$.

2. [8; 2 points each part] Suppose that you are given nonzero vectors **a**, **b**, and **c** in $\mathbf{R}^3$. Use dot and cross products to give expressions for vectors satisfying the following geometric descriptions.

a. A vector orthogonal to **a** and **b**.

Take the cross product $\mathbf{a} \times \mathbf{b}$.

b. A vector of length 2 orthogonal to **a** and **b**.

Note the lengths of vectors can be described in terms of dot products, $\|\mathbf{x}\| = \sqrt{\mathbf{x} \cdot \mathbf{x}}$.

Divide that cross product in part **a** by it's length, then multiply by 2. $\dfrac{2}{\|\mathbf{a} \times \mathbf{b}\|}\, \mathbf{a} \times \mathbf{b}$.

c. The vector projection of **b** onto **a**.

That's $\mathrm{proj}_{\mathbf{a}}\mathbf{b} = \dfrac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\|^2}\, \mathbf{a}$.

d. A vector with the length of **b** in the direction of **a**.

Divide **a** by its length, then multiply by the length of **b** to get $\dfrac{\|\mathbf{b}\|}{\|\mathbf{a}\|}\, \mathbf{a}$.