



Name: _____

Math 131 Multivariate Calculus

Quiz

26 Apr 2010

Problem 1. [6] On path integrals. Consider the path parameterized by $\mathbf{x}(t) = (\cos t, \sin t, \ln t)$ for $1 \leq t \leq 5$. Write down integrals for the following. The only variable that should appear in the integrals is t . Do not evaluate the integrals.

a. Write down an integral giving the length of the path.

b. Write down a scalar line integral for the scalar field $f(x, y, z) = x^2 + y^2 + e^z$.

c. Write down a vector line integral for the vector field $\mathbf{F}(x, y, z) = z^2\mathbf{i} + xy\mathbf{j} + 2\mathbf{k}$.

Relevant formulas.

Scalar line integral:

$$\int_{\mathbf{x}} f ds = \int_a^b f(\mathbf{x}(t)) \|\mathbf{x}'(t)\| dt$$

Vector line integral:

$$\int_{\mathbf{x}} \mathbf{F} \cdot d\mathbf{s} = \int_a^b \mathbf{F}(\mathbf{x}(t)) \cdot \mathbf{x}'(t) dt$$

Green's theorem:

$$\oint_{\partial D} M dx + N dy = \iint_D \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dx dy.$$

Problem 2. [4] On Green's theorem.

Let D be the disk $x^2 + y^2 \leq 4$. Use Green's theorem to convert the line integral

$$\oint_{\partial D} -x^2y \, dx + xy^2 \, dy$$

into a double integral. Leave the D in the double integral; don't put in limits of integration for x and y , and don't evaluate the resulting double integral.