Section 1.2 selected answers

Math 131 Multivariate Calculus
D Joyce, Spring 2014

1–7 odd, 13, 15, 17, 35, 44.

15. Give parametric equations for the line in $\mathbf{R}^2$ through the point $(2, -1)$ that is parallel to the vector $\mathbf{i} - 7\mathbf{j}$.

If you want the point to be at $\mathbf{a} = (2, 1)$ at time $t = 0$ in the direction $\mathbf{b} = (1, -7)$, then take the parametric equation to be

$$\mathbf{x}(t) = \mathbf{a} + t\mathbf{b}.$$

17. Give parametric equations for the line in $\mathbf{R}^3$ through the points $(1, 4, 5)$ and $(2, 4, -1)$.

If you want the point to be at $\mathbf{a}$ at time $t = 0$ and at $\mathbf{b}$ at time $t = 1$, take the parametric equation to be

$$\mathbf{x}(t) = \mathbf{a} + t(\mathbf{b} - \mathbf{a}).$$

35. Find the points of intersection of the line $x = 2t - 3$, $y = 3t + 2$, and $z = 5 - t$ with each of the coordinate planes $x = 0$, $y = 0$, and $z = 0$.

The line $\mathbf{x} = (2t-3, 3t+2, 5-t)$ meets the plane $x = 0$ when $2t - 3 = 0$, that is, when $t = \frac{3}{2}$. Then $y = 3t + 2 = \frac{13}{2}$, and $z = 5 - t = \frac{7}{2}$. Therefore the point of intersection is $(x, y, z)(\frac{3}{2}) = (0, \frac{13}{2}, \frac{7}{2})$. The other intersection points are found similarly.

44. **(a)** Find the distance from the point $(-2, 1, 5)$ to any point on the line $x = 3t - 5$, $y = 1 - t$, $z = 4t + 7$. Your answer should be in terms of the parameter t.

The distance from $(-2, 1, 5)$ to $(3t-5, 1-t, 4t+7)$ is

$$\sqrt{(3t-3)^2 + (-t)^2 + (4t+2)^2}$$

which simplifies to $\sqrt{25t^2 - 2t + 13}$.

(b) Now find the distance between the point $(-2, 1, 5)$ and the line $x = 3t - 5$, $y = 1 - t$, $z = 4t + 7$.

There is a nice way to do this that doesn't involve calculus, but it's not hard to do it using calculus. We want to find what value of t minimizes the function $\sqrt{25t^2 - 2t + 13}$. The same value of t will minimize the function $f(t) = 25t^2 - 2t + 13$. (That's because the square root function is an increasing function.) Since $f'(t) = 50t - 2$ is 0 when $t = \frac{1}{25}$, therefore $t = \frac{1}{25}$ gives the minimum distance, which is

$$\sqrt{25(\tfrac{1}{25})^2 - 2(\tfrac{1}{25}) + 13} = \tfrac{18}{5}.$$

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