CLARK UNIVERSITY 1887

Section 1.4 selected answers

Math 131 Multivariate Calculus

D Joyce, Spring 2014

Exercises 1, 3, 5, 11–19 odd, 25

11. Find the area of the parallelogram having vertices $(1,2,3)$, $(4,-2,1)$, $(-3,1,0)$, and $(0,-3-2)$.

Choose one of the vectors as a base point, say the first, and determine the three displacement vectors to two other points.

$$\begin{aligned}\mathbf{a} &= (4,-2,1)-(1,2,3)=(3,-4,-2)\\ \mathbf{b} &= (-3,1,0)-(1,2,3)=(-4,-1,-3)\\ \mathbf{c} &= (0,-3,-2)-(1,2,3)=(-1,-5,-5)\end{aligned}$$

The first two of these vectors are along the sides of the parallelogram while the third, being their sum, goes across the parallelogram. The length of the cross product $\mathbf{a}\times\mathbf{b}$ will give the area of the parallelogram. You'll get $\sqrt{750}$.

13. If $(\mathbf{a}\times\mathbf{b})\cdot\mathbf{c}=0$, what can you say about the geometric relation between $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$?

One thing you can say is the the volume of the parallelepiped with edges $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ is 0. That means that the three vectors all lie in a plane, that is, they're coplanar. Using the terminology from linear algebra, you can say that the three vectors are dependent.

15. Compute the area of the triangle determined by the vectors $\mathbf{a}=(1,-2,6)$ and $\mathbf{b}=(4,3,-1)$.

The area of the triangle is half the area of the parallelogram with those two vectors on its sides, so $\frac{1}{2}\|\mathbf{a}\times\mathbf{b}\|$ will do it. You'll get $\frac{1}{2}\sqrt{1002}$.

17. Compute the area of the triangle having vertices $(1,0,1)$, $(0,2,3)$, and $(-1,5,-2)$.

Choose one of the vertices to be the base point, and let $\mathbf{a}$ and $\mathbf{b}$ be the displacement vectors from it to the other two vectors. Then the area of the triangle will be $\frac{1}{2}\|\mathbf{a}\times\mathbf{b}\|$.

25. Suppose that you are given nonzero vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ in $\mathbf{R}^3$. Use dot and cross products to give expressions for vectors satisfying the following geometric descriptions.

a. A vector orthogonal to $\mathbf{a}$ and $\mathbf{b}$.

Take the cross product $\mathbf{a}\times\mathbf{b}$.

b. A vector of length 2 orthogonal to $\mathbf{a}$ and $\mathbf{b}$.

Divide that cross product in part **a** by it's length, then multiply by 2. $\dfrac{2}{\|\mathbf{a}\times\mathbf{b}\|}\,\mathbf{a}\times\mathbf{b}$.

c. The vector projection of $\mathbf{b}$ onto $\mathbf{a}$.

That's $\text{proj}_{\mathbf{a}}\mathbf{b}=\dfrac{\mathbf{a}\cdot\mathbf{b}}{\|\mathbf{a}\|^2}\,\mathbf{a}$.

d. A vector with the length of $\mathbf{b}$ in the direction of $\mathbf{a}$.

Divide $\mathbf{a}$ by its length, then multiply by the length of $\mathbf{b}$ to get $\dfrac{\|\mathbf{b}\|}{\|\mathbf{a}\|}\,\mathbf{a}$.

e. A vector orthogonal to $\mathbf{a}$ and $\mathbf{b}\times\mathbf{c}$.

Take the cross product $\mathbf{a}\times(\mathbf{b}\times\mathbf{c})$.

f. A vector in the plane determined by $\mathbf{a}$ and $\mathbf{b}$ and perpendicular to $\mathbf{c}$.

Any vector $\perp$ to $\mathbf{a}\times\mathbf{b}$ will lie in the plane determined by $\mathbf{a}$ and $\mathbf{b}$. Therefore, $(\mathbf{a}\times\mathbf{b})\times\mathbf{c}$ does the trick.

Math 131 Home Page at
`http://math.clarku.edu/~djoyce/ma131/`