CLARK UNIVERSITY 1887

Section 2.3 selected answers

Math 131 Multivariate Calculus

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Exercises 1–7, 12–14, 19–21, 29, 30, 34–36.

For the first few exercises, compute the partial derivatives.

2. $f(x,y) = e^{x^2+y^2}$.

Use the chain rule to compute the partials.

$$\frac{\partial f}{\partial x} = 2xe^{x^2+y^2}$$
$$\frac{\partial f}{\partial y} = 2ye^{x^2+y^2}$$

4. $f(x,y) = \dfrac{x^3 - y^2}{1 + x^2 + 3y^4}$.

Use the quotient rule.

$$\frac{\partial f}{\partial x} = \frac{3x^2(1+x^2+3y^4) - (x^3-y^2)2x}{(1+x^2+3y^4)^2}$$
$$\frac{\partial f}{\partial y} = \frac{-2y(1+x^2+3y^4) - (x^3-y^2)12y^3}{(1+x^2+3y^4)^2}$$

6. $f(x,y) = \ln(x^2+y^2)$.

Chain rule again.

$$\frac{\partial f}{\partial x} = \frac{2x}{x^2+y^2}$$
$$\frac{\partial f}{\partial y} = \frac{2y}{x^2+y^2}$$

12. $F(x,y,z) = xyz$.

$$\frac{\partial F}{\partial x} = yz$$
$$\frac{\partial F}{\partial y} = xz$$
$$\frac{\partial F}{\partial z} = xy$$

14. $F(x,y,z) = e^{ax}\cos by + e^{az}\sin bx$.

$$\frac{\partial F}{\partial x} = ae^{ax}\cos by + be^{az}\cos bx$$
$$\frac{\partial F}{\partial y} = -be^{ax}\sin by$$
$$\frac{\partial F}{\partial z} = ae^{az}\sin bx$$

20. Given $f(x,y,z) = \sin(xyz)$, find the gradient $\nabla f(\pi, 0, \frac{\pi}{2})$.

Since

$$\nabla f = (yz\cos xyz, xz\cos xyz, xy\cos xyz),$$

therefore

$$\nabla f(\pi, 0, \tfrac{\pi}{2}) = (0, \pi^2/2, 0)$$

29. Given $\mathbf{f}(x,y,z) = \left(xyz, \sqrt{x^2+y^2+z^2}\right)$, find the matrix $D\mathbf{f}(\mathbf{a})$ of partial derivatives where $\mathbf{a} = (1, 0, -2)$.

Since f is a function from $\mathbf{R}^3$ to $\mathbf{R}^2$, therefore $D\mathbf{f}$ will be the 2×3 matrix

$$D\mathbf{f} = \begin{bmatrix} \frac{d}{dx}xyz & \frac{d}{dy}xyz & \frac{d}{dz}xyz \\ \frac{d}{dx}\sqrt{x^2+y^2+z^2} & \frac{d}{dy}\sqrt{x^2+y^2+z^2} & \frac{d}{dz}\sqrt{x^2+y^2+z^2} \end{bmatrix}$$
$$= \begin{bmatrix} yz & xz & xy \\ \frac{x}{\sqrt{x^2+y^2+z^2}} & \frac{y}{\sqrt{x^2+y^2+z^2}} & \frac{z}{\sqrt{x^2+y^2+z^2}} \end{bmatrix}$$

30. Given $\mathbf{f}(t) = (t, \cos 2t, \sin 5t)$, find the matrix $D\mathbf{f}(0)$ of partial derivatives.

Since

$$D\mathbf{f} = \begin{bmatrix} \frac{d}{dt}\,t \\ \frac{d}{dt}\cos 2t \\ \frac{d}{dt}\sin 5t \end{bmatrix} = \begin{bmatrix} 1 \\ -2\sin 2t \\ 5\cos 5t \end{bmatrix},$$

therefore

$$D\mathbf{f}(0) = \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}.$$

34. Explain why $f(x,y) = xy - 7x^8y^2 + \cos x$ is differentiable at every point in its domain.

One of the theorems we have says that if a function $f : \mathbf{R}^n \to \mathbf{R}$ has continuous partial derivatives, then it's differentiable. This one does, so it's differentiable.

36. Explain why

$$\mathbf{f}(x,y) = \left(\frac{xy^2}{x^2+y^4}, \frac{x}{y} + \frac{y}{x} \right)$$

is differentiable at every point in its domain.

Note that the domain excludes the x-axis and the y-axis.

A function $\mathbf{f} : \mathbf{R}^n \to \mathbf{R}^m$ is differentiable if all its coordinate functions are differentiable. The two coordinate functions are differentiable for the reason given in exercise 26, that is, both partial derivatives of each are continuous. So, in effect, $\mathbf{f}$ is differentiable because all four of its partials are continuous.

Math 131 Home Page at
http://math.clarku.edu/~djoyce/ma131/