CLARK UNIVERSITY 1887

Section 5.1 selected answers

Math 131 Multivariate Calculus

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Exercises 1, 2, 3, 6, 9.

1. Evaluate $\int_0^2 \int_1^3 (x^2+y)\,dy\,dx$.

First evaluate the inner integral.

$$\begin{aligned}\int_1^3 (x^2+y)\,dy &= x^2y + \frac{y^2}{2}\Big|_{y=1}^{3} \\ &= (3x^2+\tfrac{9}{2}) - (x^2+\tfrac{1}{2}) \\ &= 2x^2+4\end{aligned}$$

Substitute that value in for the inner integral and finish the exercise.

$$\int_0^2 (2x^2+4)\,dx = \frac{2}{3}x^3 + 4x\Big|_{x=0}^{2} = \frac{40}{3}$$

2. Evaluate $\int_0^\pi \int_1^2 y\sin x\,dy\,dx$.

The inner integral is

$$\begin{aligned}\int_1^2 y\sin x\,dy &= \sin x \int_1^2 y\,dy \\ &= (\sin x)\tfrac{1}{2}y^2\Big|_1^2 = \tfrac{3}{2}\sin x\end{aligned}$$

Therefore,

$$\begin{aligned}\int_0^\pi \int_1^2 y\sin x\,dy\,dx &= \int_0^\pi \tfrac{3}{2}\sin x\,dx \\ &= -\tfrac{3}{2}\cos x\Big|_0^\pi \\ &= -\tfrac{3}{2}\cos\pi + \tfrac{3}{2}\cos 0 \\ &= 3\end{aligned}$$

3.

$$\begin{aligned}\int_{-2}^4 \int_0^1 xe^y\,dy\,dx &= \int_{-2}^4 x \int_0^1 e^y\,dy\,dx \\ &= \int_{-2}^4 x\left(e^y\Big|_{y=0}^1\right)dx \\ &= \int_{-2}^4 x(e-1)\,dx \\ &= \frac{e-1}{2}x^2\Big|_{-2}^4 \\ &= \frac{e-1}{2}(16-4) \\ &= 6e-6\end{aligned}$$

6. $\int_1^9 \int_1^e \frac{\ln\sqrt{x}}{xy}\,dx\,dy$.

First note that $\ln\sqrt{x} = \frac{1}{2}\ln x$, so that

$$\begin{aligned}\int_1^9 \int_1^e \frac{\ln\sqrt{x}}{xy}\,dx\,dy &= \int_1^9 \int_1^e \frac{\frac{1}{2}\ln x}{xy}\,dx\,dy \\ &= \int_1^9 \frac{1}{2y} \int_1^e \frac{\ln x}{x}\,dx\,dy\end{aligned}$$

Evaluate the inner integral with the substitution $u=\ln x$, $du = \frac{1}{x}dx$.

$$\int_{x=1}^e \frac{\ln x}{x}\,dx = \int_{u=0}^1 u\,du = \frac{u^2}{2}\Big|_{u=0}^1 = \frac{1}{2}$$

Therefore,

$$\begin{aligned}\int_1^9 \frac{1}{2y}\int_1^e \frac{\ln x}{x}\,dx\,dy &= \tfrac{1}{4}\int_1^9 \frac{1}{y}\,dy \\ &= \tfrac{1}{4}\ln y\Big|_1^9 \\ &= \tfrac{1}{4}\ln 9 = \tfrac{1}{2}\ln 3\end{aligned}$$

9. Find the volume of the region bounded by the graph of $f(x,y) = 2x^2 + y^4\sin\pi x$, the (x,y)-plane, and the planes $x=0, x=1, y=-1, y=2$.

The volume equals the integral

$$\int_0^1 \int_{-1}^2 |f(x,y)|\,dy\,dx.$$

(You could also interchange the order of integration and finish the problem that way.)

The first question is: is $f(x,y)$ positive when $0 \leq x \leq 1$ and $-1 \leq y \leq 2$? Yes, since sin is positive for angles between 0 and π. So, we can drop the absolute value symbol.

$$\int_0^1 \int_{-1}^2 (2x^2 + y^4 \sin \pi x)\, dy\, dx$$

First, evaluate the inner integral.

$$\begin{aligned} & \int_{-1}^2 (2x^2 + y^4 \sin \pi x)\, dy \\ = \;& 2x^2 y - \tfrac{1}{5} y^5 \sin \pi x \,\Big|_{y=-1}^2 \\ = \;& \left(4x^2 - \tfrac{32}{5} \sin \pi x\right) - \left(-2x^2 + \tfrac{1}{5} \sin \pi x\right) \\ = \;& 6x^2 - \tfrac{33}{5} \sin \pi x \end{aligned}$$

Then the original integral equals

$$\begin{aligned} & \int_0^1 (6x^2 - \tfrac{33}{5} \sin \pi x)\, dx \\ = \;& 2x^3 + \frac{33}{5\pi} \cos \pi x \Big|_0^1 \\ = \;& (2 + \tfrac{33}{5\pi} \cos \pi) - (\tfrac{33}{5\pi} \cos 0) \\ = \;& 2 - \tfrac{66}{5\pi} \end{aligned}$$

Math 131 Home Page at
`http://math.clarku.edu/~djoyce/ma131/`