CLARK

Name: ___________________________

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Math 131 Multivariate Calculus

Sample First Test

You may refer to one sheet of notes on this test. Points for each problem are in square brackets.

Problem 1. [12] On functions of several variables.

a. Give an example of a function $\mathbf{f} : \mathbf{R}^3 \to \mathbf{R}^2$.

b. Give an example of a scalar-valued function whose domain is the set

$$\{(x, y) \in \mathbf{R}^2 | (x, y) \neq (0, 0)\}.$$

c. Determine the domain of the function $f(x, y) = \dfrac{\sqrt{x-10}}{y+5}$.

Problem 2. [10] Consider the function $z = f(x, y) = y - x^2$. Draw either the level curve or the contour curve for f at height $c = 2$. (Draw one or the other. It's your choice.)

Problem 3. [20] On limits. Evaluate the limit, or explain why it fails to exist.

a. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2 + y^2}$

b. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3 + xy^2 + 2x^2 + 2y^2}{x^2 + y^2}$

Problem 4. [32] On derivatives.

a. [5] Find $\dfrac{\partial f}{\partial y}$ if $f(x, y) = (x + e^x) \sin y$.

b. [5] Find $\dfrac{\partial^2 f}{\partial x \partial y}$ for the function f given in part a.

c. [10] Compute the gradient ∇f if $f(x, y, z) = (2x + 3y + 4z)^2$.

d. [12] Find the derivative $D\mathbf{f}$ if $\mathbf{f}(x, y) = (2x + 3y, xy, \cos x)$.

Problem 5. [17] On the chain rule. Suppose that $f : \mathbf{R}^2 \to \mathbf{R}^3$ has the derivative

$$D\mathbf{f}(x, y) = \begin{bmatrix} 2xy & x^2 \\ 1 & 2y \\ y \cos xy & x \cos xy \end{bmatrix}$$

and $\mathbf{x} : \mathbf{R} \to \mathbf{R}^2$ has the derivative $D\mathbf{f}(t) = \begin{bmatrix} 3t^2 \\ e^t \end{bmatrix}$.

a. [5] The derivative $D(\mathbf{f} \circ \mathbf{x})(t)$ is a matrix. What size is that matrix?

b. [12] Find the derivative $D(\mathbf{f} \circ \mathbf{x})(t)$.

Problem 6. [10] On differentiability. Consider the function whose graph $z = f(x, y)$ is displayed below. It is defined in terms of absolute values by

$$f(x, y) = |(|x| - |y|)| - |x| - |y|.$$

As you can see from the graph, its partial derivatives evaluated at the origin are both 0, that is, $f_x(0, 0) = f_y(0, 0) = 0$. Explain why this function is not differentiable at the origin even though it's partial derivatives are both 0.

#1.[12]	
#2.[10]	
#3.[20]	
#4.[32]	
#5.[17]	
#6.[10]	
Total	