Name: _____

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Math 131 Multivariate Calculus

Sample Second Test

You may refer to one sheet of notes on this test. Points for each problem are in square brackets.

Problem 1. [15] Suppose that a scalar-valued function $f : \mathbf{R}^3 \to \mathbf{R}$ has the gradient

$$\nabla f(x, y, z) = \left(\frac{2x}{x^2 + y^2 + z^2}, \frac{2y}{x^2 + y^2 + z^2}, \frac{2z}{x^2 + y^2 + z^2} \right).$$

Compute the directional derivative $D_{\mathbf{u}}(\mathbf{a})$ in the direction $\mathbf{u} = \left(\frac{6}{7}, \frac{3}{7}, \frac{2}{7}\right)$ at the point $\mathbf{a} = (1, 2, -1)$.

Problem 2. [20] Calculate the velocity, speed, acceleration, and unit tangent vector of the path $\mathbf{x}(t) = (\cos t, \sin t, e^t)$.

a. Velocity.

b. Speed.

c. Acceleration.

d. Unit tangent vector.

Problem 3. [15] Set up an integral that gives the length of the path $\mathbf{x}(t) = (t^2, 1, 2 \ln t)$ for $1 \leq t \leq 5$. Do not evaluate the integral.

Problem 4. [20] Consider the function $f(x, y) = e^{-y}(x^2 - y^2)$. Its first and second partial derivatives are

$$f_x = 2xe^{-y} \qquad f_y = -e^{-y}(x^2 + 2y - y^2)$$

$$f_{xx} = 2e^{-y} \qquad f_{xy} = -2xe^{-y} \qquad f_{yy} = e^{-y}(x^2 + 4y - y^2 - 2)$$

a. Determine the two critical points of f.

b. Identify the nature (max, min, saddle) of each critical point.

Problem 5. [15] The vector-valued field $\mathbf{F}(x, y, z) = (y, x, -3)$ is a gradient field. Find a potential function $f : \mathbf{R}^3 \to \mathbf{R}$ for $\mathbf{F}$.

Problem 6. [15] Set up a double integral to compute the volume of a solid whose base is the plane region D in the (x, y)-plane bounded by the x-axis and the parabola $y = 4 - x^2$; and whose height at a point (x, y) in that region is given by $f(x, y) = \sin(x^2 + y^2)$. Be sure to sketch the region D. Do not evaluate the integral.

#1.[15]	
#2.[20]	
#3.[15]	
#4.[20]	
#5.[15]	
#6.[15]	
Total	