

# Math 217, Probability and Statistics

Final Exam

14 Dec 2005

Your name: \_\_\_\_\_

You may refer to two sheets of notes on this test, and you may use a calculator. You may leave your answers as expressions such as  $\binom{8}{4} \frac{e^{1/3}}{\sqrt{2\pi}}$  if you like. Show all your work for credit.

**Problem 1.** [8] Two fair dice are tossed. Let  $A$  be the event that the sum of the dice is 4, 5, or 6. Let  $B$  be the event that both dice are even. Are they independent events? Explain your answer.

**Problem 2.** [12; 6 points each part] The variance  $\sigma^2$  of a random variable  $X$  is defined as  $V(X) = V((X - \mu)^2)$  where  $\mu = E(X)$  is the mean of  $X$ .

**a.** Using properties of expectation prove that  $\sigma^2 = E(X^2) - \mu^2$ .

**b.** State the properties of expectation that you used in part (a).

**Problem 3.** [18; 6 points each part] Let  $X$  be a random variable with a moment generating function  $g_X(t)$ , mean  $\mu$ , and variance  $\sigma^2$ .

**a.** Let  $Y = -X$ . Find the moment generating function  $g_Y$  in terms of the moment generating function  $g_X$  of  $X$ .

**b.** Let  $W = X + 2$ . Find  $g_W$  in terms of  $g_X$ .

**c.** Let  $Z = \frac{X - \mu}{\sigma}$ . Find  $g_Z$  in terms of  $g_X$ .

**Problem 4.** [8] Suppose  $P(B|A) = \frac{1}{4}$ ,  $P(B|\tilde{A}) = \frac{1}{3}$ , and  $P(A) = \frac{2}{5}$ . Find  $P(A|B)$ .

**Problem 5.** [15] In an opinion poll it is assumed that an unknown proportion  $p$  of the people are in favor of a proposed new law and a proportion  $1 - p$  are against it. A sample of  $n$  people is taken to obtain their opinion. The proportion  $\bar{p}$  in favor in the sample is taken as an estimate of  $p$ . Using the Central Limit Theorem, determine how large a sample will ensure that the estimate will, with probability .95, be correct to within .01.

**Problem 6.** [12] Let  $X$  and  $Y$  be two independent standard normal distributions. Recall that the probability density function for a standard normal distribution is  $\frac{1}{\sqrt{2\pi}} e^{-x^2/2}$ . Let  $Z = X + Y$ . Use the convolution formula to compute the probability density function  $f_Z(z)$  for  $Z$ .

**Problem 7.** [15] Suppose that you are dealt a hand of 13 cards from a standard deck of 52 cards.

a. [5] What is the probability that you are dealt no spades?

b. [10] What is the probability that you are dealt at least one card from each suit? Explain your reasoning.

**Problem 8.** [12; 6 points each part] Suppose that we are observing a lump of radioactive plutonium-239. The particular size of the plutonium sample has a rate of 0.4 radioactive emissions per second.

a. What is the probability that the first emission occurs in the first second?

b. What is the probability that the first emission occurs in the second second?

#1.[8]	
#2.[12]	
#3.[18]	
#4.[8]	
#5.[15]	
#6.[12]	
#7.[15]	
#8.[12]	
Total	